University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Solution of Nonlinear Equation

Secant Method

Undergraduate Leve, 3th Stage

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Lecture Outline

1.0 Introduction

2.0 Closed Methods

- 2.1 Bisection Method
- 2.2 False-Point Position Method

3.0 Open Methods

- 3.1 Newton-Raphson Method
- 3.2 Secant Method

4.0 Summary

3.2 –Secant Method

The following steps are required in estimating the root.

1. Assume two inertial values for the root for $x_{i-1}\ and\ x_i$.
2. Evaluate $f(x_i)$.
3. Calculate the estimated root (x_{i+1}) as following:

   $x_{i+1} = x_i - \frac{f(x_i)\times(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$

4. Calculate the error value, $f(x_{i+1})$, which represents the corresponding value of x_{i+1}.
5. Compare the error value, $|f(x_{i+1})|$, with the desire accuracy ($\in$).

   If $|f(x_{i+1})| \leq \in$ then x_{i+1} is the required root (accurate enough).

   Otherwise go to step 3.

Ex1: Find a root for $f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}$ with accuracy equal to $\in = 0.000001$.

Solution:

Step 1:

Assume $x_{-1} = -0.02$ $x_0 = 0.05$

Step 2:

$f(x_{i-1}) = f(-0.02) = -0.02^3 - 0.165 \times -0.02^2 + 3.993 \times 10^{-4}$ =0.000325

$f(x_i) = f(0.05) = 0.05^3 - 0.165 \times 0.05^2 + 3.993 \times 10^{-4}$ =0.000112

Step 3:

$$x_{i+1} = x_i - \frac{f(x_i)\times(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} = 0.05 - \frac{0.000112\times(0.05 \quad (-0.02))}{(0.000112 - 0.000325)} = 0.08666$$

Step 4:

$$f(x_{i+1}) = f(0.08666) = x^3 - 0.165x^2 + 3.993 \times 10^{-4} = -0.00019$$

Step 5:

$|f(x_1)| > \in \rightarrow (not\ enough\ accurat)$ go to step 3

Step #	x_i	$f(x_1)$	$f(x_{i-1})$	x_{i+1}	$f(x_{i+1})$	$\lvert f(x_r)\rvert - \in$
0	-0.02					
1	0.05	0.000112	0.000325	0.08666	-0.00019	-0.000179004
2	0.086656	-0.00019	0.000112	0.06362	-1.1E-05	-1.07014E-06
3	0.063624	-1.1E-05	-0.00019	0.06219	1.66E-06	8.33586E-06
4	0.062191	1.66E-06	-1.1E-05	0.06238	-5.4E-09	9.99459E-06

The required root is equal to x=0.06238.

Homework 5

4.0 – Summary

The roots of nonlinear equations can be estimated numerically. In general, the closed method (Bisection and False Position methods) are guaranteed for convergence, while the open methods (Newton-Raphson and Secant methods) are not guaranteed for convergence. In addition, the open methods converge faster than the closed methods.