University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Polynomial Interpolation

Gregory-Newton Polynomial Interpolation

Undergraduate Leve, 3th Stage

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Lecture Outline

3.0 – Gregory-Newton Polynomial Interpolation

This type is special version of Newton's Interpolation. This interpolation can be used only for a set of data when the values of x are equally spaced (h).

If the data set in the following table was obtained by experiments or by observation, the value corresponding to any x can be estimated as follows:

x_i	$f(x_i)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$
.	.
.	.
x_{n-1}	$f(x_{n-1})$
x_n	$f(x_n)$

$$h = x_{i+1} - x_i$$

$$r = \frac{x - x_0}{h}$$

$$f_n(x) = f(x_0) + r\Delta f_0 + r(r-1)\frac{\Delta^2 f_0}{2!} + r(r-1)(r-2)\frac{\Delta^3 f_0}{3!} + \cdots$$

$$+ r(r-1)\ldots(r-(n-1))\frac{\Delta^n f_0}{n!}$$

$\Delta f_0, \Delta^2 f_0, \Delta^3 f_0, \ldots, \Delta^n f_0$ can be calculated as following:

	$f(x_0)$	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$
x_0	$f(x_0)$			
		$f(x_1) - f(x_0)$		
x_1	$f(x_1)$		$[f(x_2) - f(x_1)] - [f(x_1) - f(x_0)]$	
		$f(x_2) - f(x_1)$		$\{[f(x_3) - f(x_2)] - [f(x_2) - f(x_1)]\} - \{[f(x_2) - f(x_1)] - [f(x_1) - f(x_0)]\}$
x_2	$f(x_2)$		$[f(x_3) - f(x_2)] - [f(x_2) - f(x_1)]$	
		$f(x_3) - f(x_2)$		
x_3	$f(x_3)$			

Ex1: Find the value of $f(1.83)$ for the data set shown:

x	y
1	0
3	1.0986
5	1.6094
7	1.9459
9	2.1972

Solution:

$$h = x_{i+1} - x_i = 3 - 1 = 2$$

$$r = \frac{x - x_0}{h} = \frac{1.83 - 1}{2} = 0.415$$

	$f(x_0)$	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$
1	**0**				
		$\mathbf{1.0986 - 0 = 1.0986}$			
3	1.0986		$\mathbf{0.3365 - 1.0986 = -0.5878}$		
		$1.6094 - 1.0986 = 0.5108$		$\mathbf{(-0.1743) - (-0.5878) = 0.4135}$	
5	1.6094		$0.3365 - 0.5108 = -0.1743$		$\mathbf{0.0891 - 0.4135 = -0.3244}$
		$1.9459 - 1.6094 = 0.3365$		$(-0.0852) - (-0.1743) = 0.0891$	
7	1.9459		$0.2513 - 0.3365 = -0.0852$		
		$2.1972 - 1.9459 = 0.2513$			
9	2.1972				

$$f_n(1.83) = 0 + 0.415 \times (1.0986) + 0.415 \times (0.415 - 1)\frac{-0.5108}{2!} + 0.415 \times (0.415 - 1)(0.415 - 2)\frac{0.4135}{3!} + 0.415 \times (0.415 - 1)(0.415 - 2)(0.415 - 3)\frac{-0.3244}{4!} = 0.5676$$

Ex2: Find the value of $f(4.12)$ for the data set shown:

x	y
0	1
1	2
2	4
3	8
4	16
5	32

Solution:

$$h = x_{i+1} - x_i = 1 - 0 = 1$$

$$r = \frac{x - x_0}{h} = \frac{4.12 - 0}{1} = 4.12$$

x	$f(x_0)$	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$	$\Delta^5 f_0$
0	1					
		$2-1=1$				
1	2		$2-1=1$			
		$4-2=2$		$2-1=1$		
2	4		$4-2=2$		2-1=1	
		$8-4=4$		$4-2=2$		2-1=1
3	8		$8-4=4$		4-2=2	
		16 − 8−= 8		٤-٨=4		
4	16		$16-8=8$			
		$32-16=16$				
5	32					

$$f_n(4.12) = 1 + 4.12 \times (1) + 4.12(4.12-1)\frac{1}{2!} + 4.12(4.12-1)(4.12-2)\frac{1}{3!}$$

$$+ 4.12(4.12-1)(4.12-2)(4.12-3)\frac{1}{3!}$$

$$+ 4.12(4.12-1)(4.12-2)(4.12-3)(4.12-4)\frac{1}{4!} = 17.3913$$

Homework 9

For the data set, shown in the following table, estimate $f(1.65)$ using Gregory-Newton Polynomial:

i	x	$f(x)$
1	1.5	2.25
2	1.6	2.56
3	1.7	2.89
4	1.8	3.24