University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Polynomial Interpolation

General Formula

Undergraduate Leve, 3th Stage

Mr. Waleed Ali Tameemi
Engineer/ College of Engineering/ Babylon University
M.Sc. Civil Engineering/ the University of Kansas/ USA

2016-2017

Lecture Outline

1.0 Introduction

2.0 Newton's Divided Difference Formula

- 2.1 Linear Interpolation
- 2.2 Quadratic Interpolation
- 2.3 General Formula

3.0 Gregory-Newton Polynomial Interpolation

4.0 Lagrange Interpolation

5.0 Summary

2.3 – General Formula

This type of interpolation can be used for any set of data and as follows:

x_i	$f(x_i)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$
.	.
.	.
x_{n-1}	$f(x_{n-1})$
x_n	$f(x_n)$

$$f_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0)(x - x_1) \times \ldots$$
$$\times (x - x_{n-1})$$

Where:

$$b_0 = f(x_0)$$

$$b_1 = f(x_{1,}, x_0)$$

$$b_2 = f(x_{2,}, x_{1,}, x_0)$$

$$b_n = f(x_{n,}, x_{n-1,}, \dots, x_0)$$

$b_0, b_1, b_2, \dots, b_n$ can be calculated as following:

b_0		b_1	b_2	b_3
x_0	$f(x_0)$			
		$\frac{f(x_0)-f(x_1)}{x_0-x_1}$		
x_1	$f(x_1)$		$\frac{\frac{f(x_0)-f(x_1)}{x_0-x_1}-\frac{f(x_1)-f(x_2)}{x_1-x_2}}{x_0-x_2}$	
		$\frac{f(x_1)-f(x_2)}{x_1-x_2}$		$\frac{\frac{\frac{f(x_0)-f(x_1)}{x_0-x_1}-\frac{f(x_1)-f(x_2)}{x_1-x_2}}{x_0-x_2}-\frac{\frac{f(x_1)-f(x_2)}{x_1-x_2}-\frac{f(x_2)-f(x_3)}{x_2-x_3}}{x_1-x_3}}{x_0-x_3}$
x_2	$f(x_2)$		$\frac{\frac{f(x_1)-f(x_2)}{x_1-x_2}-\frac{f(x_2)-f(x_3)}{x_2-x_3}}{x_1-x_3}$	
		$\frac{f(x_2)-f(x_3)}{x_2-x_3}$		
x_3	$f(x_3)$			

Ex1: The data in Table 1 was obtained by observation, estimate the value of y at x=2.

x	y
0.5	-0.693
1	0
2.718	1
7.388	2

Solution:

$$f_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0)(x - x_1) \times \ldots$$

$$\times (x - x_{n-1})$$

b_0	b_1	b_2	b_3
0.5	**-0.693**		
	$\frac{(-0.693)-0}{0.5-1}=1.386$		
1	0	$\frac{1.386-0.582}{0.5-2.718}=-0.362$	
	$\frac{0-1}{1-2.718}=0.582$		$\frac{-0.362-(-0.0576)}{0.5-7.388}=0.0442$
2.718	1	$\frac{0.582-0.214}{1-7.388}=-0.0576$	
	$\frac{1-2}{2.718-7.388}=0.214$		
7.388	2		

$$f_3(2) = (-0.693) + 1.386(2-0.5) + (-0.362)(2-0.5)(2-1)$$
$$+ (0.0442)(2-0.5)(2-1)(2-2.718) = 0.795$$

Ex2: If the temperature (T) of a lake is given in the following table as a function of the lake depth (d), estimate the lake temperature at d = -7.5 m.

d (m)	T (^{o}C)
0	19.1
-1	19.1
-2	19.0
-3	18.8
-4	18.7
-5	18.3
-6	18.2
-7	17.6
-8	11.7
-9	9.9
-10	9.1

Solution:

For simplicity let's do third order polynomial:

Choose only four points (from -6 to -9)

$$f_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0)(x - x_1) \times \ldots$$

$$\times (x - x_{n-1})$$

b_0	b_1	b_2	b_3
-6	18.2		
	$\frac{\mathbf{18.2 - 17.6}}{\mathbf{(-6) - (-7)}} = \mathbf{0.6}$		
-7	17.6	$\frac{\mathbf{0.6 - 5.9}}{\mathbf{(-6) - (-8)}} = \mathbf{-2.65}$	
	$\frac{17.6 - 11.7}{(-7) - (-8)} = 5.9$		$\frac{\mathbf{(-2.65) - 2.05}}{\mathbf{(-6) - (-9)}} = \mathbf{-1.567}$
-8	11.7	$\frac{5.9 - 1.8}{(-7) - (-9)} = 2.05$	
	$\frac{11.7 - 9.9}{(-8) - (-9)} = 1.8$		
-9	9.9		

$$f_3(-7.5) = 18.2 + 0.6\big(-7.5 - (-6)\big) + (-2.65)\big(-7.5 - (-6)\big)\big(-7.5 - (-7)\big) + (-1.567)\big(-7.5 - (-6)\big)\big(-7.5 - (-7)\big)\big(-7.5 - (-8)\big) = 14.725\ ^{o}\text{C}$$

Ex3: The velocity (v) of a rocket is given in the following table as a function of the time (t), estimate the rocket velocity when t =16 seconds.

t (s)	v (m/s)

0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Solution:

For simplicity let's do third order polynomial:

Choose only four points (from 10 to 22.5)

$$f_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0)(x - x_1) \times \ldots$$

$$\times (x - x_{n-1})$$

b_0		b_1	b_2	b_3
10	227.04			
		$\mathbf{\frac{227.04 - 362.78}{10 - 15} = 27.148}$		
15	362.78		$\mathbf{\frac{27.148 - 30.914}{10 - 20} = 0.377}$	
		$\frac{362.78 - 517.35}{15 - 20} = 30.914$		$\mathbf{\frac{0.377 - 0.445}{10 - 22.5} = 0.005}$
20	517.35		$\frac{30.914 - 34.248}{15 - 22.5} = 0.445$	
		$\frac{517.35 - 602.97}{20 - 22.5} = 34.248$		
22.5	602.97			

$$f_n(16) = 227.04 + 27.148(16 - 10) + 0.377(16 - 10)(16 - 15)$$

$$+ 0.005(16 - 10)(16 - 15)(16 - 20) = 392.07\ m/s$$

Homework 8

For the data set, shown in the following table, estimate $f(3)$ using:

1- Third degree polynomial.

2- Fourth degree polynomial.

i	x	$f(x)$
0	-1	13
1	1	15
2	2	13
3	4	33
4	5	64