University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Solution of Nonlinear Equation

False-Point Position Method

Undergraduate Leve, 3^{th} Stage

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Lecture Outline

2.2 – False-Point Position Method

The following steps are required in estimating the equation root.

1. Assume two x-values(x_p, x_n) such that one of them gives positive y-value ($f(x_p) = y_p$) and the other gives negative y-value ($f(x_n) = y_n$).

$$f(x_p) \times f(x_n) < 0$$

2. Calculate the estimated root (x_r) as following:

$$x_r = x_n - \frac{f(x_n) \times (x_p - x_n)}{f(x_p) - f(x_n)}$$

3. Calculate the error value, $|f(x_r)|$, which represents the corresponding value of x_r.
4. Compare the error value, $|f(x_r)|$, with the desire accuracy ($\in$).
5. If $|f(x_r)| \leq \in$ then x_r is the required root (accurate enough).

   If $|f(x_r)| > \in$ and $f(x_r)$ is positive then $x_p = x_r$ and go to step 2.

   If $|f(x_r)| > \in$ and $f(x_r)$ is negative then $x_n = x_r$ and go to step 2.

Ex1: Find a root for $f(x) = e^{-x} - x$ with accuracy equal to $\in = 0.001$.

Solution:

Step 1:

Assume

$x_p = 0$ $\rightarrow$ $f(0) = e^{-0} - 0 = 1$

$x_n = 1 \quad \rightarrow \quad f(1) = e^{-1} - 1 = -0.632$

Step 2:

$$x_r = x_n - \frac{f(x_n) \times (x_p - x_n)}{f(x_p) - f(x_n)} \quad \rightarrow \quad x_r = 1 - \frac{-0.632 \times (0 - 1)}{1 - (-0.632)} = 0.6127$$

Step 3:

$f(x_r) = f(0.6127) = e^{-0.6127} - 0.6127 = -0.0708$

Step 4:

$|f(x_r)| = |-0.0708| = 0.0708 > \in = 0.001 \rightarrow (not\ enough\ accurat)$

Step 5:

Since $f(x_r) = -0.0708$ (negative)

Then $x_n = x_r = 0.6127 \quad \rightarrow f(0.6127) = e^{-0.6127} - 0.6127 = -0.0708$

$x_p = 0 \quad \rightarrow \quad f(0) = e^{-0} - 0 = 1$ (from last step)

Start again from step 2

Step #	x_p	$f(x_p)$	x_n	$f(x_n)$	x_r	$f(x_r)$	$\lvert f(x_r)\rvert - \in$
1	0	1	1	-0.63212	0.6127	-0.07081	-0.06981
2	0.5	0.106531	0.6127	-0.07081	0.567699	-0.00087	0.00013

The required root is equal to $x_r = 0.5676$.

Ex2: Find a root for $f(x) = \cos x$ with accuracy equal to $\in= 0.00001$.

Note: All measurements in this example are in radian.

Solution:

Step 1:

Assume

$x_p = 2 \quad \rightarrow \quad f(2) = \sin 2 = 0.90929$

$x_n = 4 \quad \rightarrow \quad f(4) = \sin 4 = -0.75680$

Step 2:

$$x_r = x_n - \frac{f(x_n) \times (x_p - x_n)}{f(x_p) - f(x_n)} \quad \rightarrow \quad x_r = 1 - \frac{-0.75680 \times (2-4)}{0.90929 - (-0.75680)}$$

$$= 3.0915$$

Step 3:

$f(x_r) = f(3.0915) = \sin 3.0915 = 0.050044$

Step 4:

$|f(x_r)| = |0.050044| = 0.050044 > \in = 0.00001 \rightarrow (not\ enough\ accurat)$

Step 5:

Since $f(x_r) = 0.050044$ (positive)

Then $x_p = x_r = 3.0915 \quad\quad \rightarrow f(3.0915) = 0.050044$

$x_n = 4 \quad\quad \rightarrow \quad\quad f(4) = \sin 4 = -0.75680$ (from last step)

Start again from step 2

Step #	x_p	$f(x_p)$	x_n	$f(x_n)$	x_r	$f(x_r)$	$\|f(x_r)\| - \in$
1	2	0.909297	4	-0.7568	3.091528	0.050044	-0.05003
2	3.091528	0.050044	4	-0.7568	3.147875	-0.00628	-0.00627
3	3.091528	0.050044	3.147875	-0.00628	3.14159	2.3E-06	7.7E-06

The required root is equal to $x_r = 3.1415$.

Ex3: Find a root for $f(x) = x^3 + x^2 - 3x - 3$ with accuracy equal to $\in = 0.0001$.

Solution:

Step 1:

Assume

$x_p = 2 \quad\quad \rightarrow \quad\quad f(2) = 2^3 + 2^2 - 3 \times 2 - 3 = 3$

$x_n = 1 \quad\quad \rightarrow \quad\quad f(1) = 1^3 + 1^2 - 3 \times 1 - 3 = -4$

Step 2:

$$x_r = x_n - \frac{f(x_n) \times (x_p - x_n)}{f(x_p) - f(x_n)} \quad \rightarrow \quad x_r = 1 - \frac{-4 \times (2-1)}{3 - (-4)} = 1.57142$$

Step 3:

$$f(x_r) = f(1.57142) = 1.57142^3 + 1.57142^2 - 3 \times 1.57142 - 3 = -1.36449$$

Step 4:

$$|f(x_r)| = |-1.36449| = 1.36449 > \in = 0.0001 \ \rightarrow (not\ enough\ accurat)$$

Step 5:

Since $f(x_r) = -1.36449$ (negative)

Then $x_n = x_r = 1.57142 \qquad \rightarrow f(1.57142) = -1.36449$

$x_p = 2 \qquad \rightarrow \qquad f(2) = 2^3 + 2^2 - 3 \times 2 - 3 = 3$

Start again from step 2

Step #	x_p	$f(x_p)$	x_n	$f(x_n)$	x_r	$f(x_r)$	$\|f(x_r)\| - \in$
1	1	-4	2	3	1.571429	-1.36443	-1.36433
2	1	-4	1.571429	-1.36443	1.867257	1.395343	-1.39524
3	1.867257	1.395343	1.571429	-1.36443	1.717686	-0.13468	-0.13458
4	1.867257	1.395343	1.717686	-0.13468	1.730851	-0.01134	-0.01124
5	1.867257	1.395343	1.730851	-0.01134	1.731951	-0.00094	-0.00084
6	1.867257	1.395343	1.731951	-0.00094	1.732043	-7.8E-05	2.19E-05

The required root is equal to $x_r = 1.7320$.

Homework 2

1- Find a root for $f(x) = x^3 - 2x^2$ with accuracy equal to $\in= 0.01$.

Start with: $x_p = 3$ and $x_n = 1$.

2- Find a root for $f(x) = \cos(x + 1)$ with accuracy equal to $\in= 0.0001$.

Start with: $x_p = 1$ and $x_n = 0.5$.