University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Polynomial Interpolation

Quadratic Interpolation

Undergraduate Leve, 3th Stage

Mr. Waleed Ali Tameemi
Engineer/ College of Engineering/ Babylon University
M.Sc. Civil Engineering/ the University of Kansas/ USA

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Lecture Outline

1.0 Introduction

2.0 Newton's Divided Difference Formula

- 2.1 Linear Interpolation
- 2.2 Quadratic Interpolation
- 2.3 General Formula

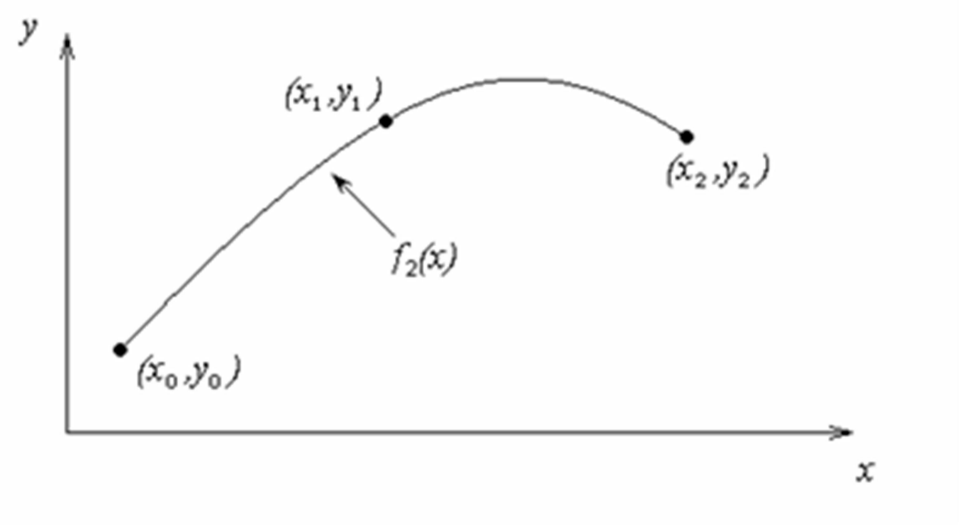
3.0 Gregory-Newton Polynomial Interpolation

4.0 Lagrange Interpolation

5.0 Summary

2.2 – Quadratic Interpolation

If three points, $(x_0, f(x_0))$, $\big(x_1, f(x_1)\big), and\ (x_2, f(x_2))$, are given and we asked to evaluate the value of $f_2(x)$ corresponding to any x that located between $(x_0, f(x_0))$ and $\big(x_2, f(x_2)\big)$ and near $\big(x_1, f(x_1)\big)$, as shown in Figure 2.



$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

Where:

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Ex1: The data in Table 1 was obtained by observation, estimate the value of y at x=2.

x	y

1	0
2.718	1
5.92	2

Solution:

By quadratic interpolation, x=2

$b_0 = f(x_0) = 0$

$$b_1 = \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right) = \frac{1-0}{2.718-1} = 0.58$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} = \frac{\frac{2-1}{5.92-2.718} - 0.58}{5.92-1} = -0.05$$

$$f_2(2) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$= 0 + 0.58(2-1) + (-0.05)(2-1)(2-2.718) = 0.62$$

Ex2: If the temperature (T) of a lake is given in the following table as a function of the lake depth (d), estimate the lake temperature at d = -7.5 m.

d (m)	T (^{o}C)
0	19.1
-1	19.1
-2	19.0
-3	18.8
-4	18.7
-5	18.3
-6	18.2
-7	17.6
-8	11.7
-9	9.9

-10	9.1

Solution:

By quadratic interpolation, d = -7.5m

Choose (-7,17.6), (-8,11.7), and (-9,9.9) because they are located around d = -7.5m

$b_0 = f(x_0) = f(-7) = 17.6$

$$b_1 = \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right) = \left(\frac{11.7 - 17.6}{(-8) - (-7)}\right) = 5.9$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} = \frac{\frac{9.9 - 11.7}{(-9) - (-8)} - 5.9}{(-9) - (-7)} = 2.05$$

$f_2(-7.5) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) = 17.6 + 5.9((-7.5) -$

$(-7)) + (2.05)((-7.5) - (-7))((-7.5) - (-8)) = 14.138\ ^{o}C$

Ex3: The velocity (v) of a rocket is given in the following table as a function of the time (t), estimate the rocket velocity when t =16 seconds.

t (s)	v (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Solution:

By quadratic interpolation, t = 16 m/s

Choose (10 , 227.04), (15 , 362.78), and (20 , 517.35) because they are located around t = 16 m/s

$$b_0 = f(x_0) = f(10) = 227.04$$

$$b_1 = \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right) = \left(\frac{362.78 - 227.04}{15 - 10}\right) = 27.148$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} = \frac{\frac{517.35 - 362.78}{20 - 15} - 27.148}{20 - 10} = 0.3766$$

$$f_2(16) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$= 227.04 + 27.148(16 - 10) + 0.3766(16 - 10)(16 - 15)$$

$$= 392.19 m/s$$

Homework 7

For the data set, shown in the following table, estimate $f(3)$ using:

1- Quadratic polynomial (Second Degree Polynomial).

i	x	$f(x)$
0	-1	13
1	1	15
2	2	13
3	4	33
4	5	64