Transpose of matrix

The transpose of an mxn matrix A is the nxm matrix denoted by A^T , formed by interchanging the rows and columns of A the ith rows of A is the ith columns in A^T.

For Example: $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix}_{2\times3}$ $A^T = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -1 \end{pmatrix}_{3\times2}$

9 – Symmetric Matrix: A square matrix A such that $A = A^T$ is called symmetric matrix i.e. A is a symmetric matrix if and only if aij = a_{ji} for all element.

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

For Example: a $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, b

10 – Skew symmetric Matrix: A square matrix A such that $A = A^T$ is called that A is skew symmetric matrix. i.e A is skew matrix $\longleftrightarrow$ a_{ji} = -aij for all element of A.

The following are examples of symmetric and skew – symmetric matrices respectively

$$(a)\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}, (b)\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$$

(a) symmetric

(b) Skew – symmetric.

Note the fact that the main diagonal element of a skew – symmetric matrix must all be Zero

11 – Determinates: To every square matrix that is assigned a specific number called the determinates of the matrix.

(a) Determinates of order one: write det (A) or $|A|$ for detrimental of the matrix A. it is a number assigned to square matrix only.

The determinant of (1×1) matrix (a) is the number a itself det (a) = a.

(c) Determinants of order two: the determinant of the 2×2. matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Is denoted and defined as follows: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Theorem 1.2: determinant of a product of matrices is the product of the determinant of the matrices is the product of the determinant of the matrices det (A B) = det (A). det (B) det (A + B) # det a + det B

(C) Determinates of order three:

(i) the determinant of matrix is defined as follows:

$$\begin{vmatrix} + & - & + \\ a11 & a22 & a23 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{vmatrix} = a11 \begin{vmatrix} a22 & a23 \\ a32 & a33 \end{vmatrix} - a12 \begin{vmatrix} a21 & a23 \\ a31 & a33 \end{vmatrix} + a13 \begin{vmatrix} a21 & a22 \\ a31 & a32 \end{vmatrix}$$

(ii) Consider the (3×3) matrix $\begin{vmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \\ & & \end{vmatrix}$

= a11 a22 a33 + a21 a22 a31 + a13 a21 a32

- a31 a22 a13 - a32 a23 a11 -a33 a21 a12.

Show that the diagram papering below where the first two columns are rewritten to the right of the matrix.

Theorem 1.3:

A matrix is invertible if and only if its determinant is <u>not Zero</u> usually a matrix is said to be singular if determinant is zero and non singular it otherwise.

1.5 prosperities of Determinants

(1) det A = det A^T where A^T is the transpose of A.

(2) if any two rows (or two columns) of a determinates are interchanged the value of determinants is multiplied by -1.

(3) if all elements in row (or column) of a square matrix are zero.

Then det (A) = 0

(4) if two parallel column (rows) of square matrix A are equal then det (A) = 0

(5) if all the elements of one row (or one column) of a determinant are multiplied by the same factor K. the value of the new determinant is K times the given det.

Example;

$$\begin{pmatrix} 4 & 6 & 1 \\ 3 & -9 & 2 \\ -1 & 12 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 2.3 & 1 \\ 3 & -3.3 & 2 \\ -1 & 4.3 & 3 \end{pmatrix}$$

$$= 3\begin{pmatrix} 4 & 2 & 1 \\ 3 & -3 & 2 \\ -1 & 4 & 3 \end{pmatrix}$$

Example: $\begin{pmatrix} 1 & 0 & 4 \\ -2 & 5 & -8 \\ 3 & 6 & 12 \end{pmatrix} = 4\begin{pmatrix} 1 & 0 & 1 \\ -2 & 5 & -2 \\ 3 & 6 & 3 \end{pmatrix} = 0$

(6) if to each element of a selected row (or column) of a square matrix = k times. The corresponding element of another selected row (or column) is added.

Example: $\begin{vmatrix} 2 & 0 & 2 \\ 1 & -1 & +1 \\ 3 & 0 & 2 \end{vmatrix} = -1\begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} = 2$

$2\times$ row (1) + row (3) $\begin{vmatrix} 2 & 0 & 2 \\ 1 & -1 & 1 \\ 7 & 0 & 6 \end{vmatrix} = -1\begin{vmatrix} 2 & 2 \\ 7 & 6 \end{vmatrix} = 2$

(7) if any row or column contain zero elements and only one element not zero then the determinant will reduced by elementary the row and column if the specified element indeterminate.