Consider an arbitrary system of equation in unknown as:

AX = B ……………………………………………………………(1)

$$\left.\begin{array}{l} a_{r1}X_1 + a_{12}X_2 + a_B X_3 + ... + a_{1n}X_n \\ ail\chi_1 + ai_2\chi_2 + ai_3\chi 3 + ... + ain\chi n = b_1 \\ a21\chi_1 + a22\chi 2 + a23\chi 3 + ... + a2n\chi n = b2 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + ... + a_{2n}X_n \\ am1\chi 1 + am2X2 + 9m3\chi 3 + ... + a2n\chi n = bm \\ a_{m1}X_1 + a_{m2}X_2 + a_{m3}X_3 + ... + a_{mn}X_n \end{array}\right\} \text{…………………………(2)}$$

The coefficient of the variables and constant terms can be put in the form:

$$\begin{pmatrix} a_{11} & a_{12} & a_{1n} \\ a2 & a22 & a2n \\ a_{m1} & a_{m2} & a_{mn} \end{pmatrix}_{mxn} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \\ \chi n \end{pmatrix}_{nx1} \begin{pmatrix} b1 \\ b2 \\ \\ bm \end{pmatrix}_{mx1} \text{……………………………………..(3)}$$

Let the form

$$\begin{pmatrix} a_u a_{12} a_{1n} \\ \\ a_{21} a_{22} a_{2n} \\ \\ am_1 am_2 amn \end{pmatrix} = A = (a_{i1}) \text{……………………………………………..(4)}$$

Is called (mxn) matrix and donated this matrix by:

[aij] i = 1, 2,……m and j = 1, 2,…..n.

We say that is an (mxn) matrix or

The matrix of order (mxn) it has m rows and n columns.

For example the first row is (a_{11}, a12, a_{1n})

And the first column is $\begin{pmatrix} a_{12} \\ a_{11} \\ a_{21} \\ a_{m1} \end{pmatrix}$

(aij) denote the element of matrix. Lying in the i – th row and j – th column, and we call this element as the (i,j) - th element of this matrix

Also $\begin{pmatrix} \chi 1 \\ \chi_2 \\ \\ \chi_n \end{pmatrix}_{nx1}$ is (nx1) [n rows and columns]

$\begin{pmatrix} b1 \\ b2 \\ \\ bm \end{pmatrix}_{mx}$ Is (mx1) [m rows and 1 column]

Sub – Matrix:

Let A be matrix in (4) then the sub-matrix of A is another matrix of A denoted by deleting rows and (or) column of A.

Let A = $\begin{pmatrix} 1\ 2\ 3 \\ 4\ 5\ 6 \\ 7\ 8\ 9 \end{pmatrix}$

Find the sub-matrix of A with order (2×3) any sub-matrix of A denoted by deleting any row of A $\begin{pmatrix}1\,2\,3\\4\,5\,6\end{pmatrix}, \begin{pmatrix}1\,2\,3\\7\,8\,9\end{pmatrix}, \begin{pmatrix}4\,5\,6\\7\,8\,9\end{pmatrix}$

Definition 1.1**:**

Tow (mxn) matrices A = [aij] (mxn) and B = [bij] (mxn) are said to be equal if and only if:

aij = bij for i = 1,2…..m and j = 1,2….n

Thus two matrices are equal if and only if:

i. They have the same dimension, and
ii. All their corresponding elements are equal for example:

$$\begin{bmatrix}2 & 0 - 1\\3 & 5 \quad 2\end{bmatrix} = \begin{bmatrix}\frac{4}{2} & 0(7) & -2+1\\3 & \frac{20}{4} & 2\end{bmatrix}$$

Definition 1.2

If A = [aij] mxn and B = [bij] mxn are mxn matrix their sum is the mxn matrix A+B = $[aij + bij]mxn$.

In other words if two matrices have the same dimension, they may be added by addition corresponding elements. For example if:

A = $\begin{pmatrix}2 & -7\\-3 & 4\end{pmatrix}$ *and* **B**= $\begin{pmatrix}-5 & 0\\1 & 6\end{pmatrix}$

Then

A+B = $\begin{pmatrix} 3+-5 & -7+0 \\ -3+1 & 4+6 \end{pmatrix} = \begin{pmatrix} -3 & -7 \\ -2 & 10 \end{pmatrix}$

Additions of matrices, like equality of matrices is defined only of matrices have same dimension.

Theorem 1.1:

Addition of matrices is commutative and associative, that is if A, B and C are matrices having the same dimension then:

A + B = B + A (commutative)

A + (b + C) = (A + B) + C (associative)

Definition

The product of a scalar K and an mxn matrix A = [aij] mxn is the nn,Xn matrix KA = [kaij] mXn for example:

$$6\begin{pmatrix} -1 & 0 & 7 \\ 5 & 2 & -11 \end{pmatrix} = \begin{pmatrix} 6(-1) & 6(0) & 6(7) \\ 6(5) & 6(2) & 6(-11) \end{pmatrix} = \begin{pmatrix} -6 & 0 & 2 \\ 30 & 12 & -6 \end{pmatrix}$$

Application of Matrices

Definition :

If A = [aij] mxn is mxn matrix and B = [bjk] nxp an nxp matrix, the product AB is the mxp matrix C = [cik] nxp in which

$$Cik = \sum_{j-1}^{n} aij\, bik$$

Example1: if A = $\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{pmatrix}_{2x3}$ $and\ B = \begin{pmatrix} b11 \\ b21 \\ b22 \end{pmatrix}_{3\times1}$

$$A\,B = \begin{pmatrix} a11\,b11 + a12\,b21 + a13\,b31 \\ a21\,b11 + a22\,b21 + a23\,b31 \end{pmatrix}_{2\times1}$$

Example 2: Let A = $\begin{pmatrix} 2 & 3 \\ -1 & 4 \\ 5 & -2 \end{pmatrix}_{3\times 2}$ $and\ B = \begin{pmatrix} 3 & 1 & 4 & -5 \\ -2 & 0 & 3 & 4 \end{pmatrix}$

A B = $\begin{pmatrix} 0 & 2 & 17 & 2 \\ -11 & -1 & 8 & 21 \\ 19 & 5 & 14 & -33 \end{pmatrix}_{3\times 4}$

Note 1.1:

1 – in general if A and B are two matrices. Then A B may not be equal of

BA. For example A = $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} and\ B\,A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

$\therefore A\,B \neq B\,A$

2 – if A B is defined then its not necessary that B A must also be defined.

For example. If A is of order (2×3) and B of order (3×1) then clearly A B is define, but B A is not defined.

1.3 Different Types of matrices:

1 – Row Matrix: A matrix which has exactly one row is called row matrix.

For example (1, 2, 3, 4) is row matrix

2 – Column Matrix: A matrix which has exactly one column is called a

column matrix for example $\begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$ is a column matrix.

3 – Square Matrix**:** A matrix in which the number of row is equal to the number of columns is called a square matrix for example $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is a 2×2 square matrix.

A matrix (A) $(n \times n)$ A is said to be order or to be an n-square matrix.

4 - Null or Zero Matrix**:** A matrix each of whose elements is zero is called null matrix or zero matrix, for example $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is a (2×3) null matrix.

5 – Diagonal Matrix: the elements aii are called diagonal of a square matrix (a_{11} a_{22} – a_{nn}) constitute its main diagonal A square matrix whose every element other than diagonal elements is zero is called a diagonal matrix for

Example: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

6 – Scalar Matrix: A diagonal matrix, whose diagonal elements are equal, is called a scalar matrix.

For example $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ are scalar matrix

7 – Identity Matrix: A diagonal matrix whose diagonal elements are all equal to 1 (unity) is called identity matrix or (unit matrix). And denoted by in for

Example $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Note1.2: if A is (mxn) matrix, it is easily to define that AIn = A and also ImA = A

Ex: Find AI and IA when $A = \begin{pmatrix} 3 & 7 & 2 \\ 1 & -1 & 3 \end{pmatrix}$

Solution: IA $\longrightarrow$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times2} \begin{pmatrix} 3 & 7 & 2 \\ 1 & -4 & 3 \end{pmatrix}_{2\times3} = \begin{pmatrix} 3 & 7 & 2 \\ 1 & -4 & 3 \end{pmatrix}_{2\times3}$

And $AI = \begin{pmatrix} 3 & 7 & 2 \\ 1 & -4 & 3 \end{pmatrix}_{2\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3} = \begin{pmatrix} 3 & 7 & 2 \\ 1 & -4 & 3 \end{pmatrix}_{2\times3}$

8 – Triangular Matrix: A square matrix (aij) whose element aij = 0 whenever $j \langle i$ is Called a lower triangular matrix.simillary y a square matrix (aij) whose element aij = 0 whenever is called an upper Tringular Matrix

For example: $\begin{pmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ are lower triangular matrix

And

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$, $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$, are upper triangular