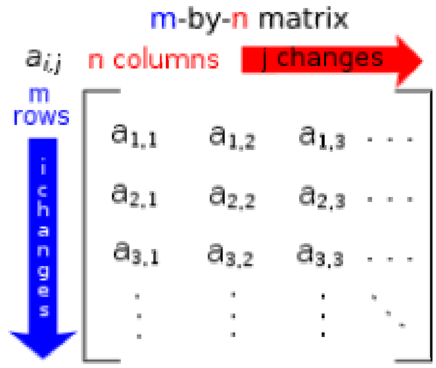


A *matrix* is a rectangular array of numbers or other mathematical objects, for which operations such as addition and multiplication are defined.[5] Most commonly, a matrix over a field F is a rectangular array of scalars from F.[6][7] Most of this article focuses on *real* and *complex matrices*, i.e., matrices whose elements are real numbers or complex numbers, respectively. More general types of entries are discussed below. For instance, this is a real matrix:

$$\mathbf{A} = \begin{bmatrix} -1.3 & 0.6 \\ 20.4 & 5.5 \\ 9.7 & -6.2 \end{bmatrix}.$$

The numbers, symbols or expressions in the matrix are called its *entries* or its *elements*. The horizontal and vertical lines of entries in a matrix are called *rows* and *columns*, respectively.

Size

The size of a matrix is defined by the number of rows and columns that it contains. A matrix with m rows and n columns is called an $m \times n$ matrix or m-by-n matrix, while m and n are called its *dimensions*. For example, the matrix **A** above is a 3×2 matrix.

Matrices which have a single row are called *row vectors*, and those which have a single column are called *column vectors*. A matrix which has the same number of rows and columns is called a *square matrix*. A matrix with an infinite number of rows or columns (or both) is called an *infinite matrix*. In some contexts, such as computer algebra programs, it is useful to consider a matrix with no rows or no columns, called an *empty matrix*.

Name	Size	Example	Description
Row vector	$1 \times n$	$\begin{bmatrix} 3 & 7 & 2 \end{bmatrix}$	A matrix with one row, sometimes used to represent a vector

Column vector	$n \times 1$	$\begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$	A matrix with one column, sometimes used to represent a vector
Square matrix	$n \times n$	$\begin{bmatrix} 9 & 13 & 5 \\ 1 & 11 & 7 \\ 2 & 6 & 3 \end{bmatrix}$	A matrix with the same number of rows and columns, sometimes used to represent a linear transformation from a vector space to itself, such as reflection, rotation, or shearing.

Notation

Matrices are commonly written in box brackets:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

An alternative notation uses large parentheses instead of box brackets:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

The specifics of symbolic matrix notation varies widely, with some prevailing trends. Matrices are usually symbolized using upper-case letters (such as **A** in the examples above), while the corresponding lower-case letters, with two subscript indices (e.g., a_{11}, or $a_{1,1}$), represent the entries. In addition to using upper-case letters to symbolize matrices, many authors use a special typographical style, commonly boldface upright (non-italic), to further distinguish matrices from other mathematical objects. An alternative notation involves the use of a double-underline with the variable name, with or without boldface style, (e.g., $\underline{\underline{A}}$).

The entry in the i-th row and j-th column of a matrix **A** is sometimes referred to as the i,j, (i,j), or $(i,j)^{\text{th}}$ entry of the matrix, and most commonly denoted as $a_{i,j}$, or a_{ij}. Alternative notations for that entry are $A[i,j]$ or $A_{i,j}$. For example, the (1,3) entry of the following matrix **A** is 5 (also denoted a_{13}, $a_{1,3}$, $A[1,3]$ or $A_{1,3}$):

$$\mathbf{A} = \begin{bmatrix} 4 & -7 & 5 & 0 \\ -2 & 0 & 11 & 8 \\ 19 & 1 & -3 & 12 \end{bmatrix}$$

Sometimes, the entries of a matrix can be defined by a formula such as $a_{i,j} = f(i, j)$. For example, each of the entries of the following matrix **A** is determined by $a_{ij} = i - j$.

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

In this case, the matrix itself is sometimes defined by that formula, within square brackets or double parenthesis. For example, the matrix above is defined as $\mathbf{A} = [i\text{-}j]$, or $\mathbf{A} = ((i\text{-}j))$. If matrix size is $m \times n$, the above-mentioned formula $f(i, j)$ is valid for any $i = 1, ..., m$ and any $j = 1, ..., n$. This can be either specified separately, or using $m \times n$ as a subscript. For instance, the matrix **A** above is 3×4 and can be defined as $\mathbf{A} = [i - j]$ ($i = 1, 2, 3$; $j = 1, ..., 4$), or $\mathbf{A} = [i - j]_{3\times4}$.

Some programming languages utilize doubly subscripted arrays (or arrays of arrays) to represent an m-×-n matrix. Some programming languages start the numbering of array indexes at zero, in which case the entries of an m-by-n matrix are indexed by $0 \leq i \leq m - 1$ and $0 \leq j \leq n - 1$.[8] This article follows the more common convention in mathematical writing where enumeration starts from 1.

The set of all m-by-n matrices is denoted $\mathbb{M}(m, n)$.

Basic operations

There are a number of basic operations that can be applied to modify matrices, called *matrix addition*, *scalar multiplication*, *transposition*, *matrix multiplication*, *row operations*, and *submatrix*.[10]

Addition, scalar multiplication and transposition

Main articles: Matrix addition, Scalar multiplication and Transpose

Operation	Definition	Example

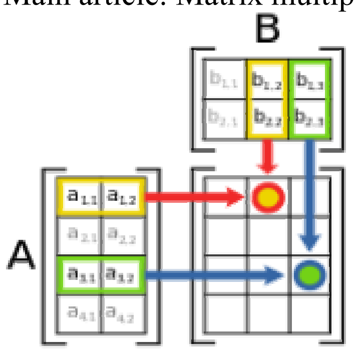
Addition	The *sum* **A**+**B** of two *m*-by-*n* matrices **A** and **B** is calculated entrywise: $(\mathbf{A} + \mathbf{B})_{i,j} = \mathbf{A}_{i,j} + \mathbf{B}_{i,j}$, where $1 \le i \le m$ and $1 \le j \le n$.	$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 1+5 \\ 1+7 & 0+5 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 8 & 5 & 0 \end{bmatrix}$
Scalar multiplication	The product *c***A** of a number *c* (also called a scalar in the parlance of abstract algebra) and a matrix **A** is computed by multiplying every entry of **A** by *c*: $(c\mathbf{A})_{i,j} = c \cdot \mathbf{A}_{i,j}$. This operation is called *scalar multiplication*, but its result is not named “scalar product” to avoid confusion, since “scalar product” is sometimes used as a synonym for “inner product”.	$2 \cdot \begin{bmatrix} 1 & 8 & -3 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 & 2\cdot 8 & 2\cdot -3 \\ 2\cdot 4 & 2\cdot -2 & 2\cdot 5 \end{bmatrix} = \begin{bmatrix} 2 & 16 & -6 \\ 8 & -4 & 10 \end{bmatrix}$

Transposition	The *transpose* of an *m*-by-*n* matrix **A** is the *n*-by-*m* matrix $\mathbf{A}^{\mathrm{T}}$ (also denoted $\mathbf{A}^{\mathrm{tr}}$ or ${}^{\mathrm{t}}\mathbf{A}$) formed by turning rows into columns and vice versa: $(\mathbf{A}^{\mathrm{T}})_{i,j} = \mathbf{A}_{j,i}$.	$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & 7 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 \\ 2 & -6 \\ 3 & 7 \end{bmatrix}$

Familiar properties of numbers extend to these operations of matrices: for example, addition is commutative, i.e., the matrix sum does not depend on the order of the summands: **A** + **B** = **B** + **A**.[11] The transpose is compatible with addition and scalar multiplication, as expressed by $(c\mathbf{A})^{\mathrm{T}} = c(\mathbf{A}^{\mathrm{T}})$ and $(\mathbf{A} + \mathbf{B})^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}} + \mathbf{B}^{\mathrm{T}}$. Finally, $(\mathbf{A}^{\mathrm{T}})^{\mathrm{T}} = \mathbf{A}$.

Matrix multiplication

Main article: Matrix multiplication



Schematic depiction of the matrix product **AB** of two matrices **A** and **B**.

Multiplication of two matrices is defined if and only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If **A** is an *m*-by-*n* matrix and **B** is an *n*-by-*p* matrix, then their *matrix product* **AB** is the *m*-by-*p* matrix whose entries are given by dot product of the corresponding row of **A** and the corresponding column of **B**:

$$[\mathbf{AB}]_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \cdots + A_{i,n}B_{n,j} = \sum_{r=1}^{n} A_{i,r}B_{r,j},$$

where $1 \le i \le m$ and $1 \le j \le p$.[12] For example, the underlined entry 2340 in the product is calculated as $(2 \times 1000) + (3 \times 100) + (4 \times 10) = 2340$:

$$\begin{bmatrix} \underline{2} & \underline{3} & \underline{4} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \underline{1000} \\ 1 & \underline{100} \\ 0 & \underline{10} \end{bmatrix} = \begin{bmatrix} 3 & \underline{2340} \\ 0 & 1000 \end{bmatrix}.$$

Matrix multiplication satisfies the rules (**AB**)**C** = **A**(**BC**) (associativity), and (**A**+**B**)**C** = **AC**+**BC** as well as **C**(**A**+**B**) = **CA**+**CB** (left and right distributivity), whenever the size of the matrices is such that the various products are defined.[13] The product **AB** may be defined without **BA** being defined, namely if **A** and **B** are m-by-n and n-by-k matrices, respectively, and $m \neq k$. Even if both products are defined, they need not be equal, i.e., generally

$$\mathbf{AB} \neq \mathbf{BA},$$

i.e., *matrix multiplication is not commutative,* in marked contrast to (rational, real, or complex) numbers whose product is independent of the order of the factors. An example of two matrices not commuting with each other is:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix},$$

whereas

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}.$$

Besides the ordinary matrix multiplication just described, there exist other less frequently used operations on matrices that can be considered forms of multiplication, such as the Hadamard product and the Kronecker product.[14] They arise in solving matrix equations such as the Sylvester equation.

Row operations

Main article: Row operations

There are three types of row operations:

1. row addition, that is adding a row to another.
2. row multiplication, that is multiplying all entries of a row by a non-zero constant;
3. row switching, that is interchanging two rows of a matrix;

These operations are used in a number of ways, including solving linear equations and finding matrix inverses.