Square matrices

A square matrix is a matrix with the same number of rows and columns. An *n*-by-*n* matrix is known as a square matrix of order *n.* Any two square matrices of the same order can be added and multiplied. The entries a_{ii} form the main diagonal of a square matrix. They lie on the imaginary line which runs from the top left corner to the bottom right corner of the matrix.

Main types

Name	**Example with *n* = 3**
Diagonal matrix	$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$
Lower triangular matrix	$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$
Upper triangular matrix	$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

Diagonal and triangular matrices

If all entries of **A** below the main diagonal are zero, **A** is called an *upper triangular matrix*. Similarly if all entries of *A* above the main diagonal are zero, **A** is called a *lower triangular matrix*. If all entries outside the main diagonal are zero, **A** is called a diagonal matrix.

Identity matrix

The identity matrix $\mathbf{I}_n$ of size *n* is the *n*-by-*n* matrix in which all the elements on the main diagonal are equal to 1 and all other elements are equal to 0, e.g.

$$I_1 = \begin{bmatrix}1\end{bmatrix},\ I_2 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \cdots,\ I_n = \begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\end{bmatrix}$$

It is a square matrix of order *n*, and also a special kind of diagonal matrix. It is called identity matrix because multiplication with it leaves a matrix unchanged:

$\mathbf{AI}_n = \mathbf{I}_m\mathbf{A} = \mathbf{A}$ for any *m*-by-*n* matrix **A**.

Symmetric or skew-symmetric matrix

A square matrix **A** that is equal to its transpose, i.e., $\mathbf{A} = \mathbf{A}^{\mathrm{T}}$, is a symmetric matrix. If instead, **A** was equal to the negative of its transpose, i.e., $\mathbf{A} = -\mathbf{A}^{\mathrm{T}}$, then **A** is a skew-symmetric matrix. In complex matrices, symmetry is often replaced by the concept of

Hermitian matrices, which satisfy $\mathbf{A}^* = \mathbf{A}$, where the star or asterisk denotes the conjugate

transpose of the matrix, i.e., the transpose of the complex conjugate of **A**.

By the spectral theorem, real symmetric matrices and complex Hermitian matrices have an eigenbasis; i.e., every vector is expressible as a linear combination of eigenvectors. In both cases, all eigenvalues are real.[25] This theorem can be generalized to infinite-dimensional situations related to matrices with infinitely many rows and columns, see below.

Invertible matrix and its inverse

A square matrix **A** is called *invertible* or *non-singular* if there exists a matrix **B** such that

$\mathbf{AB} = \mathbf{BA} = \mathbf{I}_n$.[26][27]

If **B** exists, it is unique and is called the *inverse matrix* of **A**, denoted $\mathbf{A}^{-1}$.

Orthogonal matrix

An *orthogonal matrix* is a square matrix with real entries whose columns and rows are orthogonal unit vectors (i.e., orthonormal vectors). Equivalently, a matrix A is orthogonal if its transpose is equal to its inverse:

$$A^{\mathrm{T}} = A^{-1},$$

which entails

$$A^{\mathrm{T}} A = A A^{\mathrm{T}} = I,$$

where I is the identity matrix.

An orthogonal matrix A is necessarily invertible (with inverse $A^{-1} = A^{\mathrm{T}}$), unitary ($A^{-1} = A^*$), and normal ($A^*A = AA^*$). The determinant of any orthogonal matrix is either +1 or −1. A *special orthogonal matrix* is an orthogonal matrix with determinant +1. As a linear transformation, every orthogonal matrix with determinant +1 is a pure rotation, while every orthogonal matrix with determinant -1 is either a pure reflection, or a composition of reflection and rotation.

The complex analogue of an orthogonal matrix is a unitary matrix.

Main operations

Trace

The trace, tr(**A**) of a square matrix **A** is the sum of its diagonal entries. While matrix multiplication is not commutative as mentioned above, the trace of the product of two matrices is independent of the order of the factors:

$$\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA}).$$

This is immediate from the definition of matrix multiplication:

$$\mathrm{tr}(\mathsf{AB}) = \sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij} B_{ji} = \mathrm{tr}(\mathsf{BA}).$$

Also, the trace of a matrix is equal to that of its transpose, i.e.,

$$\mathrm{tr}(\mathbf{A}) = \mathrm{tr}(\mathbf{A}^{\mathrm{T}}).$$