Physics
First class

Calculus

28/11/2016
Lecture one

1

Chapter One: Limits and Continuity

Compositions of Functions:

Given $f(x) = x^2$ and $g(x) = x+1$, the composition of f with g is $f(g(x)) = f(x+1) = (x+1)^2$.

- This composition is denoted as $f \circ g$ and reads as f composed with g.

Definition: Given two functions f and g, the composition function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$.

Example: If $f(x) = 3x-5$ and $g(x) = x+2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Sol.: $(f \circ g)(x) = f(g(x)) = f(x+2) = 3(x+2) - 5 = \boxed{3x+1}$.

$(g \circ f)(x) = g(f(x)) = g(3x-5) = (3x-5)+2 = \boxed{3x-3}$.

Example: Given $f(x) = 3x^2+2x-5$ and $g(x) = 2x-3$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Sol.: $(f \circ g)(x) = f(g(x)) = f(2x-3) = 3(2x-3)^2 + 2(2x-3) - 5$

$= 3(4x^2-12x+9) + 4x - 6 - 5 = \boxed{12x^2 - 32x + 16}$.

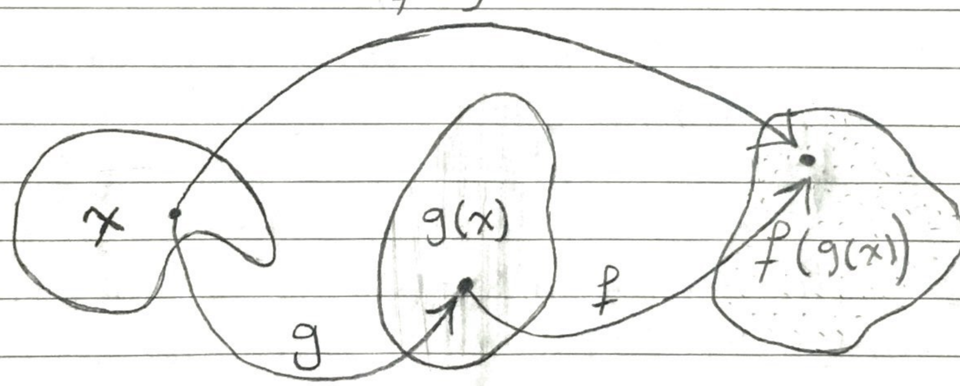
$$(g\circ f)(x) = g(f(x)) = g(3x^2+2x-5) = 2(3x^2+2x-5)-3$$

$$= \boxed{6x^2+4x-13}$$

<u>Note that</u>: In general $(f\circ g)(x) \neq (g\circ f)(x)$.

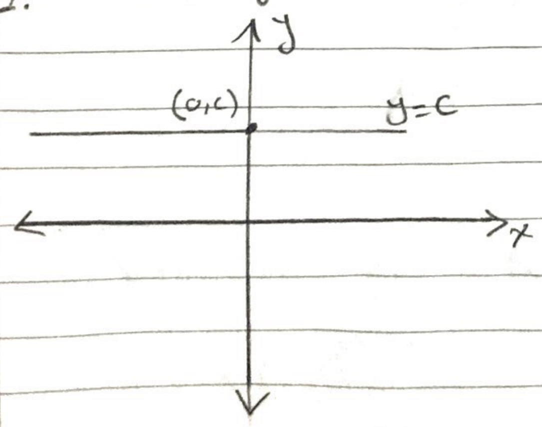
H.W.: Given $f(x) = 6x-7$ and $g(x) = x^2+3x+5$.
Find: $(g\circ f)(-1)$, $(f\circ f)(2)$, $(g\circ g)(0)$.

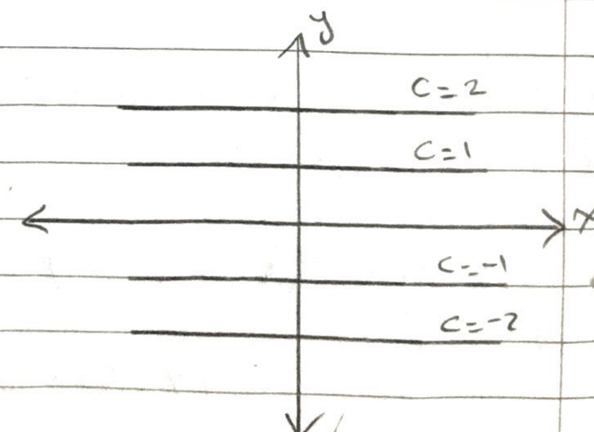
<u>Note that</u>: Compositio two functions $(f\circ g)(x)$ means:



Families of Functions:

1. Families of Curves: $y=c$



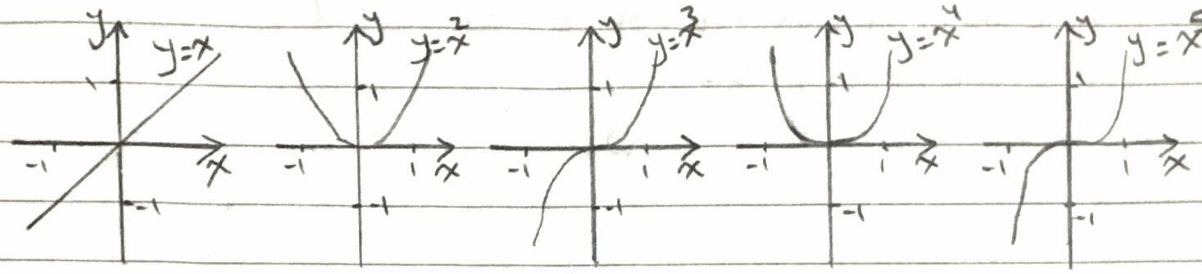


Physics
First class

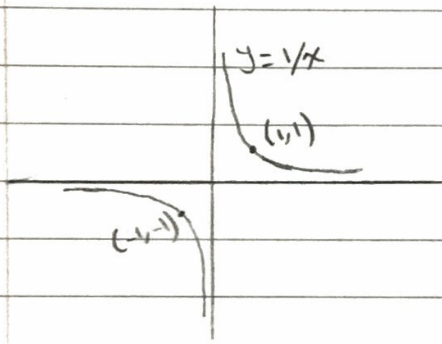
Calculus

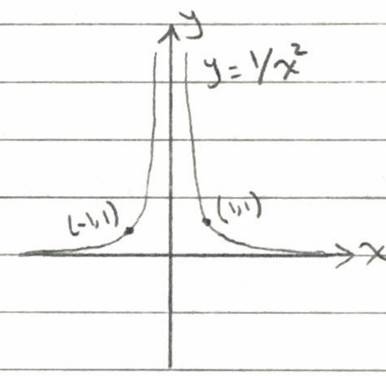
29/11/2016
Lecture one

2. Power Functions : $y = x^n$

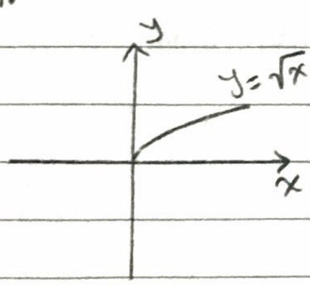


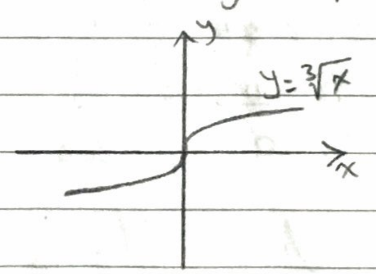
3. The family $y = x^{-n}$

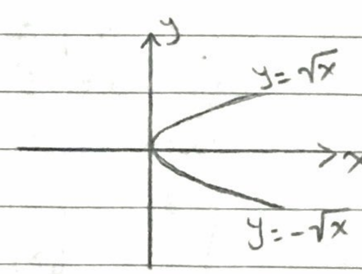




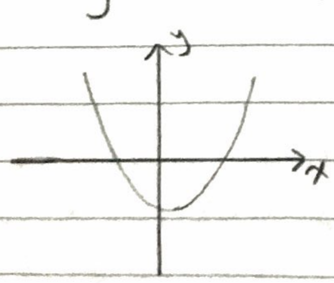
4. Power Functions with noninteger exponents : $f(x) = x^{1/n} = \sqrt[n]{x}$



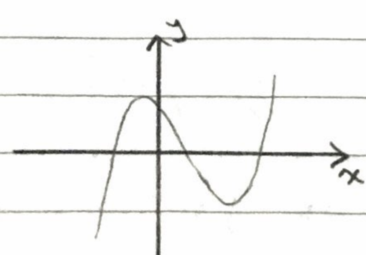




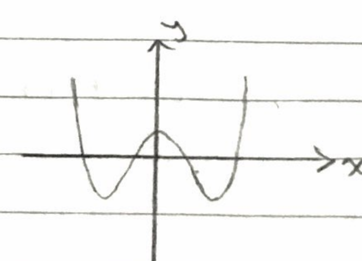
5. Polynomials : $f(x) = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$



Degree 2



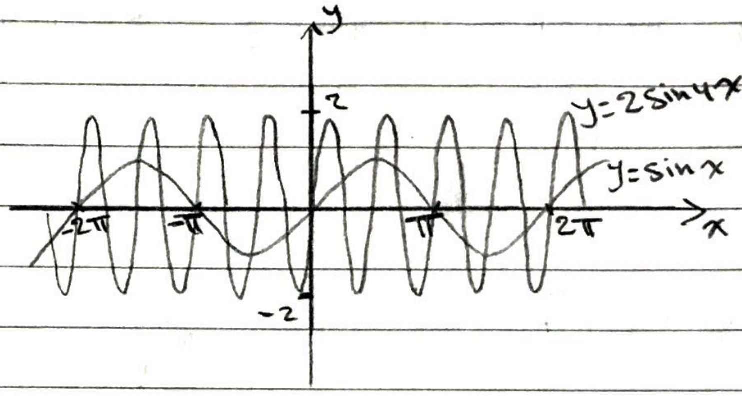
Degree 3



Degree 4

6. The families $y = A\sin Bx$ and $y = A\cos Bx$

The graph of $y = 2\sin 4x$ varies between -2 and 2, and repeats every $2\pi/4 = \pi/2$ units.



The Inverse of a Function:

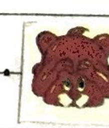
The inverse of the function f is the function that sends each $f(x)$ back to x. We denote the inverse of f by f^{-1}

Definition: If the functions f and g satisfy the two conditions

$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f$$
$$f(g(y)) = y \text{ for every } y \text{ in the domain of } g$$

then we say that f is an inverse of g and g is an inverse of f or that f and g are inverse functions.

Warning! $f^{-1}(x)$ **Never Means** $\dfrac{1}{f(x)}$.



Example: Find $f^{-1}(y)$ of $y = \sqrt{3x-2}$.

Sol.: $y = \sqrt{3x-2} \Rightarrow y^2 = 3x-2 \Rightarrow y^2+2 = 3x$

$$\Rightarrow x = \frac{y^2+2}{3} = f^{-1}(y)$$

Theorem: If an equation $y = f(x)$ can be solved for x as a function of y, say $x = g(y)$, then f has an inverse and that inverse is $g(y) = f^{-1}(y)$

Example: Find the formula for the inverse of $f(x) = 3x - 2$

Sol. $y = 3x - 2 \Rightarrow y + 2 = 3x \Rightarrow x = \frac{y+2}{3} = f^{-1}(y)$

Inverse Trigonometric Functions:

Since the definition of an inverse function says that

$f^{-1}(x) = y \Rightarrow f(y) = x$ we have the inverse sine function

$$\sin^{-1} x = y \Rightarrow \sin y = x \quad \text{and} \quad -\pi/2 \leq y \leq \pi/2$$

Example: Evaluate $\sin^{-1}(1/2)$

Sol.: we have $\sin^{-1}(1/2) = \pi/6$
because $\sin(\pi/6) = 1/2$

Definition:

Function	Domain	Range
$y = \sin^{-1} x$ if and only if $\sin y = x$	$-1 \leq x \leq 1$	$-\pi/2 \leq y \leq \pi/2$
$y = \cos^{-1} x$ if and only if $\cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$ if and only if $\tan y = x$	$-\infty \leq x \leq \infty$	$-\pi/2 \leq y \leq \pi/2$

Examples: Evaluate ① $\sin^{-1} 1$ ② $\sin^{-1} 0$ ③ $\sin^{-1} \frac{\sqrt{3}}{2}$

④ $\cos^{-1} 1$ ⑤ $\cos^{-1} 0$ ⑥ $\cos^{-1} \frac{1}{2}$

⑦ $\tan^{-1} 0$ ⑧ $\tan^{-1} \frac{1}{\sqrt{3}}$ ⑨ $\tan^{-1} \sqrt{3}$

Sol.: H.W. 

Exponential and Logarithmic Functions:

Definition: A function of the form $f(x) = b^x$, where $b > 0$, is called an <u>exponential function</u> with base b. For example:

$$f(x) = 2^x, \quad f(x) = \left(\frac{1}{2}\right)^x$$

The function $f(x) = e^x$ is called the <u>natural exponential function</u>. that base e is a certain irrational number whose value to six decimal places is $e \approx 2.718282$

Definition: We call the function $f(x) = \log_b x$ the <u>logarithmic function with base b</u>, read the logarithm to the base b of x. For example

$$\log_{10} 100 = 2, \quad \log_{10} 1/1000 = -3, \quad \log_2 16 = 4$$

$$\boxed{10^2 = 100}, \quad \boxed{10^{-3} = 1/1000}, \quad \boxed{2^4 = 16}$$

Theorem: If $b > 0$ and $b \neq 1$, then b^x and $\log_b x$ are inverse functions.

Definition: The most important logarithms in applications are those with base e. These are called <u>natural logarithms</u> because the function $\log_e x$ is the inverse of the natral exponential function e^x. It is standard to denote the natural logarithm

Calculus

of x by $\ln x$ (read ell en of x), rather than $\log_e x$.
For example

$\ln 1 = 0$, $\ln e = 1$, $\ln 1/e = -1$, $\ln e^2 = 2$

$\boxed{e^0 = 1}$, $\boxed{e^1 = e}$, $\boxed{e^{-1} = 1/e}$, $\boxed{e^2 = e^2}$

Theorem: Algebric Properties of Logarithms

If $b > 0$, $b \neq 1$, $a > 0$, $c > 0$ and r any real number then

① $\log_b (ac) = \log_b a + \log_b c$

② $\log_b (a/c) = \log_b a - \log_b c$

③ $\log_b (a^r) = r \log_b a$

④ $\log_b (1/c) = -\log_b c$

Example: Solve $4e^{1+3x} - 9e^{5-2x} = 0$

Sol.: $4e^{1+3x} = 9e^{5-2x}$

$$\frac{e^{1+3x}}{e^{5-2x}} = \frac{9}{4}$$

$$e^{1+3x-(5-2x)} = \frac{9}{4}$$

$$e^{5x-4} = \frac{9}{4}$$

$$5x - 4 = \ln\left(\frac{9}{4}\right)$$

$$5x = 4 + \ln\left(\frac{9}{4}\right)$$

$$\therefore x = \frac{1}{5}\left[4 + \ln\left(\frac{9}{4}\right)\right].$$