12th& 13th weeks

Fluid Flow Concepts and Basic Equations

Definitions:

Descriptions of Fluid Motion

To describe fluid flow patterns and to identify important characteristics of the flow field, engineer must know the following:

Streamlines and Flow Patterns

- **Flow patterns**: it is the draw or pattern which visualize the flow field.
- **The streamline***:* is a line drawn through the flow field in such a manner that the local velocity vector is tangent to the streamline at every point along the line at that instant.

Note: the tangent of the streamline at a given time gives the direction of the velocity vector.

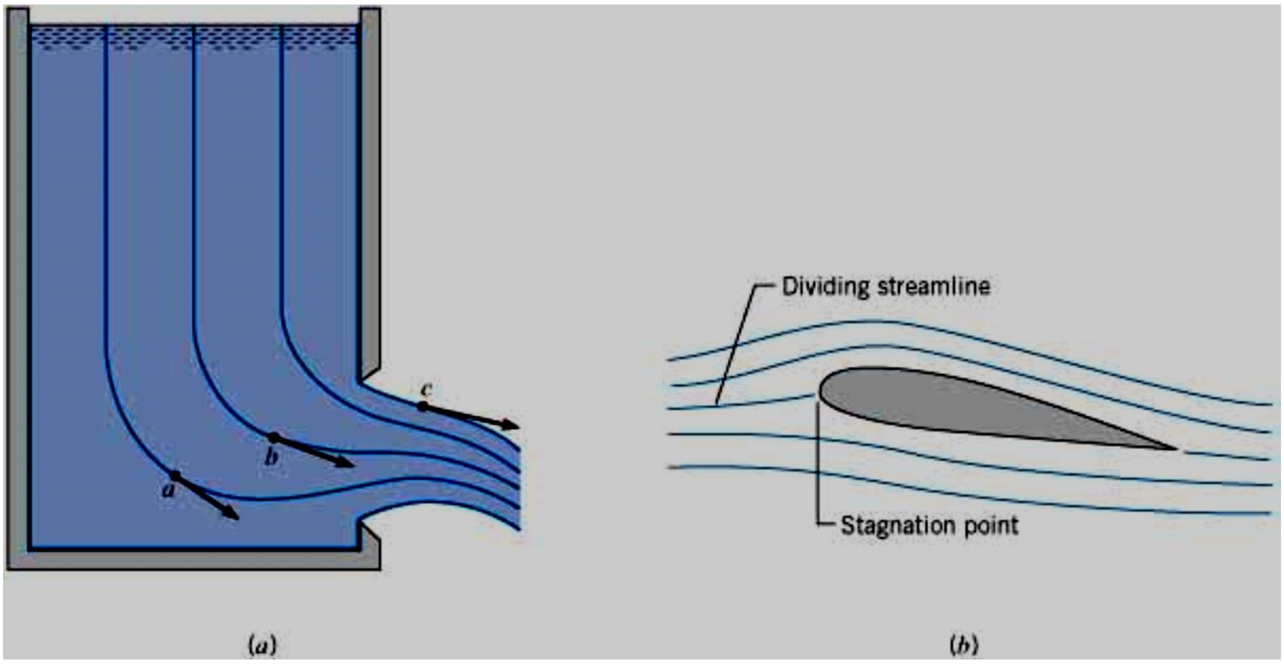


Figure 12.1 ***Flow through an opening in a tank and over an airfoil section***.

An example of streamlines and a flow pattern is shown in ***Fig. 12.1.a*** for water flowing through a slot in the side of a tank.

The velocity vectors have been sketched at three different locations: *a*, *b*, and *c*.

Note that the streamlines, according to their definition, are tangent to the velocity vectors at these points. Also, the velocities are **parallel to the wall** in the wall region, so the streamlines adjacent to the wall follow the contour of the wall.
Whenever flow occurs around a body, part of it will go to one side and part to the other as shown in ***Fig. 12.1.b*** for flow over an airfoil section.

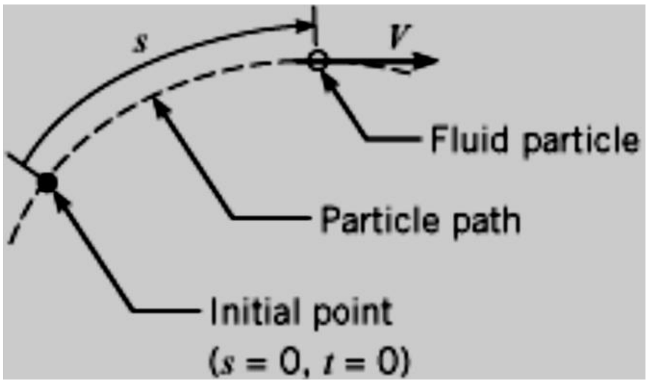


Figure 12.2 ***Fluid particle moving along a pathline***

the velocity of the fluid in the form, as shown in **Fig. 12.2**.

$$V = V(s, t)$$

Where:
s: is the distance traveled by a fluid particle along a path,
t: is the time.

Kinds of flow

Different types of flows can be:

i. **Uniform flow:**
it is the flow where the velocity does not change along a fluid path; that is, it follows that in uniform flow the fluid paths are straight and parallel as shown in **Fig. 12.3** for flow in a pipe.

$$\frac{\partial V}{\partial x} = 0$$

Figure 12.3 ***Uniform flow in a pipe***

ii. **Nonuniform flow**: the flow where the velocity changes along a fluid path, so:

$$\frac{\partial V}{\partial x} \neq 0$$

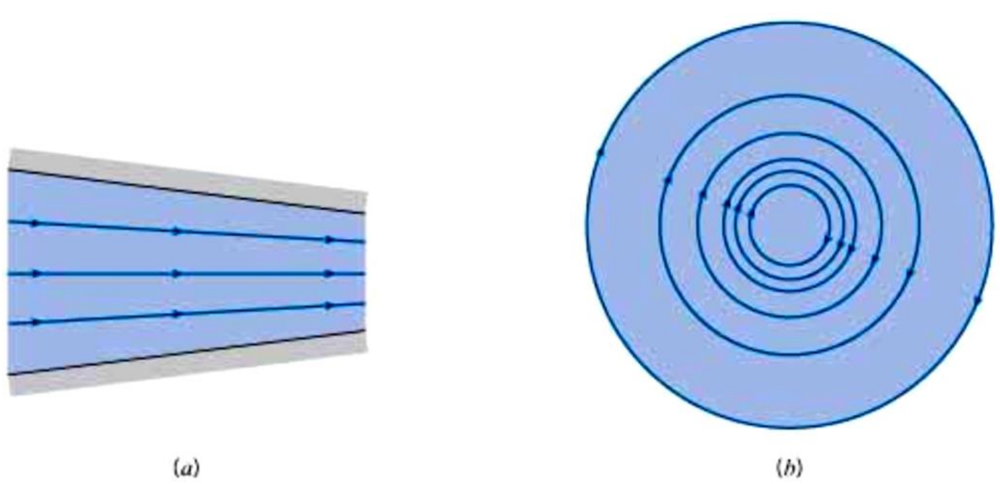


Figure 12.4:***Flow patterns for nonuniform flow. (a) Converging flow. (b) Vortex flow***.

For the converging duct in **Fig. 12.4*a***, the **magnitude of the velocity increases** as the duct converges, so the flow is nonuniform.

For the vortex flow shown in **Fig. 12.4*b***, the magnitude of the velocity does not change along the fluid path, **but the direction does**, so the flow is nonuniform.

Flows can be either :

i. **S*teady flow:*** the velocity at a given point on a fluid path does not change with time:

$$\frac{\partial V}{\partial t} = 0$$

The flow in a pipe, shown in **Fig. 12.3**, would be an example of steady flow if there was no change in velocity with time

ii. ***Unsteady flow***: . An *unsteady flow* exists if the velocity change with time.

$$\frac{\partial V}{\partial t} \neq 0$$

Note:
If the flow in the pipe changed with time due to a valve opening or closing, the flow would be **unsteady**; that is, the velocity at any point selected on a fluid path would be increasing or decreasing with time. Although ***unsteady***, the flow would still be ***uniform***.

Laminar and Turbulent Flow

Laminar flow: is a well-ordered state of flow in which adjacent fluid layers move smoothly with respect to each other.

A typical laminar flow would be the flow of honey or thick syrup from a pitcher. Laminar flow in a pipe has a smooth, parabolic velocity distribution as shown in **Fig. 12.5.a.** In general, laminar pipe flows are associated with low velocities. Laminar flows can occur in small tubes, highly viscous flows, or flows with low velocities

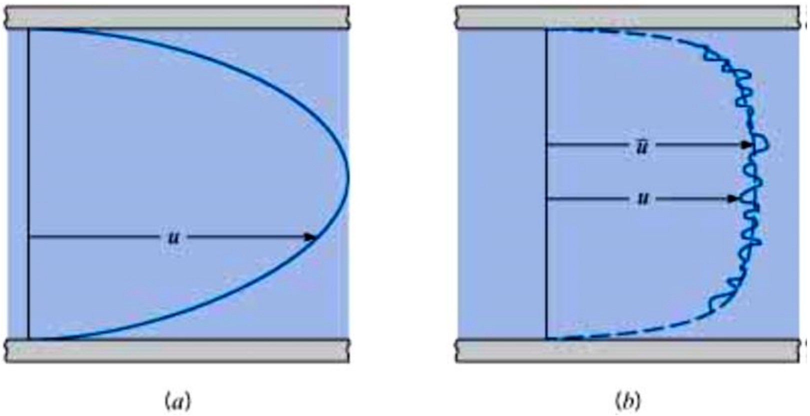


Figure 12.5 ***Laminar and turbulent flow in a straight pipe. (a) Laminar flow. (b) Turbulent flow.***

Turbulent flow : is an unsteady flow.

For example, the flow in the wake of a ship is turbulent. The eddies observed in the wake cause intense mixing. The transport of smoke from a smoke stack on a windy day also exemplifies a turbulent flow.
Turbulent pipe flows are associated with high . turbulent flows are, the most common.

One-Dimensional and Multi-Dimensional Flows

The dimensionality of a flow field is characterized by the number of spatial dimensions needed to describe the velocity field.

One-Dimensional Flow:

It is the flow where the velocity depends on only one dimension**.**

Fig. 12.6*a* shows the velocity distribution for an axisymmetric flow in a circular duct. The flow is uniform, or fully developed, so the velocity does not change in the flow direction (***z***). The velocity depends on only one dimension, namely the **radius *r***, so the flow is **one-dimensional**.

Multi-Dimensional Flows

Fig. 12.6*b* shows the velocity distribution for uniform flow in a square duct. In this case the velocity depends on two dimensions, namely *x* and *y*, so the flow is **two-dimensional.**

Figure 12.6*c* also shows the velocity distribution for the flow in a square duct but the duct cross-sectional area is expanding in the flow direction so the velocity will be dependent on ***z*** as well as ***x*** and ***y***. This flow is **three-dimensional**.

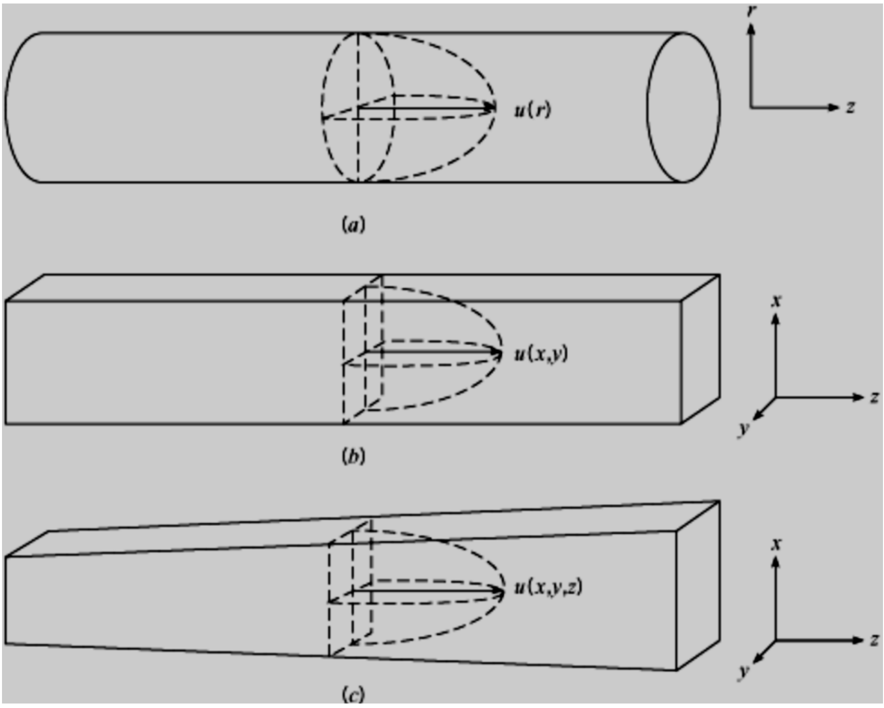


Figure 12.6: ***Flow dimensionality, (a) one-dimensional flow, (b) two-dimensional flow, and (c) three dimensional flow***.

Basic flow equations

The main task in fluid dynamics is to find the velocity field describing the flow in a given domain. To do this, one uses the basic equations of fluid flow. These encode the familiar laws of mechanics:

- **Conservation of Mass (the Continuity equation)**
- **Conservation of Energy.**
- **Conservation of Momentum (the Cauchy equation)**

Conservation of Mass (the Continuity equation)

All the basic differential equations can be derived by considering either an element. Here we choose an infinitesimal fixed control volume (*dx*, *dy*, *dz*), as in **Fig. 13. 1**. The flow through each side of the element is approximately one-dimensional, and so the appropriate mass-conservation relation to use here is

$$\int_{CV} \frac{\partial \rho}{\partial t}\, d\mathcal{V} + \sum_i (\rho_i A_i V_i)_{\text{out}} - \sum_i (\rho_i A_i V_i)_{\text{in}} = 0$$

Or:

$$\frac{D\, m_{sys}}{Dt} = \frac{D}{Dt} \int_{V_{sys}} \rho dV = 0 \qquad (5.1)$$

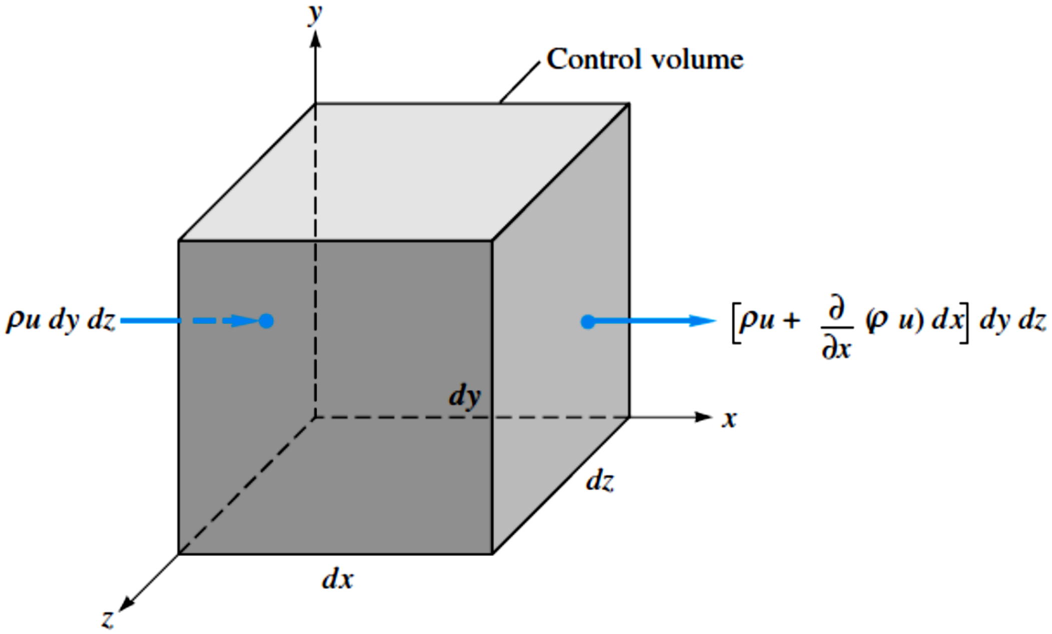


Fig. 13.1: elemental control volume

The mass-flow terms occur on all six faces, three inlets and three outlets.

Figure 4.1 shows only the mass flows on the x or left and right faces. The flows on the y (bottom and top) and the z (back and front) faces have been omitted to avoid cluttering up the drawing. We can list all these six flows as follows:

Face	Inlet mass flow	Outlet mass flow
x	$\rho u\, dy\, dz$	$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\, dx\right] dy\, dz$
y	$\rho v\, dx\, dz$	$\left[\rho v + \frac{\partial}{\partial y}(\rho v)\, dy\right] dx\, dz$
z	$\rho w\, dx\, dy$	$\left[\rho w + \frac{\partial}{\partial z}(\rho w)\, dz\right] dx\, dy$

Introducing these terms in continuity eq:

$$\frac{\partial \rho}{\partial t}\,dx\,dy\,dz + \frac{\partial}{\partial x}(\rho u)\,dx\,dy\,dz + \frac{\partial}{\partial y}(\rho v)\,dx\,dy\,dz + \frac{\partial}{\partial z}(\rho w)\,dx\,dy\,dz = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

This is the conservation of mass for an infinitesimal control volume. It is often called the ***equation of continuity***.

That is, the flow may be either steady or unsteady, viscous or frictionless, compressible or incompressible.

The vector-gradient operator:

$$\nabla = i\,\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \equiv \nabla\,(\rho\,V)$$

So:

$$\frac{\partial \rho}{\partial t} + \nabla\,(\rho\,V) = 0$$

Example 5.1:

The density changes in a pipe, due to temperature variation and other reasons, can be approximated as

$$\frac{\rho(x,t)}{\rho_0} = \left(1 - \frac{x}{L}\right)^2 \cos\frac{t}{t_0}.$$

The conduit shown in Figure 5.4 length is L and its area is A. Express the mass flow in and/or out, and the mass in the conduit as function of time. Write the expression for the mass change in the pipe.

SOLUTION

Here it is very convenient to choose a non-deformable control volume that is inside the conduit dV is chosen as $\pi\,R^2\,dx$. Using equation (5.10), the flow out (or in) is

$$\frac{d}{dt}\int_{c.v.} \rho dV = \frac{d}{dt}\int_{c.v.} \overbrace{\rho_0 \left(1 - \frac{x}{L}\right)^2 \cos\left(\frac{t}{t_0}\right)}^{\rho(t)} \overbrace{\pi\,R^2\,dx}^{dV}$$

The density is not a function of radius, r and angle, θ and they can be taken out the integral as

$$\frac{d}{dt}\int_{c.v.} \rho dV = \pi\,R^2 \frac{d}{dt}\int_{c.v.} \rho_0 \left(1 - \frac{x}{L}\right)^2 \cos\left(\frac{t}{t_0}\right) dx$$

which results in

$$\textbf{Flow Out} = \overbrace{\pi\,R^2}^{A} \frac{d}{dt}\int_0^L \rho_0 \left(1 - \frac{x}{L}\right)^2 \cos\frac{t}{t_0} dx = -\frac{\pi\,R^2\,L\,\rho_0}{3\,t_0} \sin\left(\frac{t}{t_0}\right)$$

The flow out is a function of length, L, and time, t, and is the change of the mass in the control volume.

The drawing

One dimensional flow:

Additional simplification of the continuity equation is of one dimensional flow. This simplification provides very useful description for many fluid flow phenomena. The main assumption made in this model is that the proprieties in the across section are only function of x coordinate . This assumptions leads

$$\int_{A_2} \rho_2 U_2 \, dA - \int_{A_1} \rho_1 U_1 \, dA = \frac{d}{dt} \int_{V(x)} \rho(x) \, \overbrace{A(x) \, dx}^{dV}$$

When the density can be considered constant

$$\int_{A_2} U_2 \, dA - \int_{A_1} U_1 \, dA = \frac{d}{dt} \int A(x) dx$$

For steady state but with variations of the velocity and variation of the density

$$\int_{A_2} \rho_2 U_2 \, dA = \int_{A_1} \rho_1 U_1 \, dA$$

For steady state and uniform density and velocity

$$\rho_1 A_1 U_1 = \rho_2 A_2 U_2$$

For incompressible flow (constant density), continuity equation is at its minimum form of

$$U_1 A_1 = A_2 U_2$$

Example 5.7:

Air flows into a jet engine at $5\,kg/sec$ *while fuel flow into the jet is at* $0.1\,kg/sec$. *The burned gases leaves at the exhaust which has cross area* $0.1\,m^2$ *with velocity of* $500\,m/sec$. *What is the density of the gases at the exhaust?*

Solution

The mass conservation equation (5.13) is used. Thus, the flow out is (5 + 0.1) $5.1\,kg/sec$ The density is

$$\rho = \frac{\dot{m}}{A\,U} = \frac{5.1\,kg/sec}{0.01\,m^2\;500\,m/sec} = 1.02 kg/m^3$$

Example 5.8:

The tank is filled by two valves which one filled tank in 3 hours and the second by 6 hours. The tank also has three emptying valves of 5 hours, 7 hours, and 8 hours. The tank is 3/4 fulls, calculate the time for tank reach empty or full state when all the valves are open. Is there a combination of valves that make the tank at steady state?

Solution

Easier measurement of valve flow rate can be expressed as fraction of the tank per hour. For example valve of 3 hours can be converted to 1/3 tank per hour. Thus, mass flow rate in is

$$\dot{m}_{in} = 1/3 + 1/6 = 1/2 tank/hour$$

The mass flow rate out is

$$\dot{m}_{out} = 1/5 + 1/7 + 1/8 = \frac{131}{280}$$

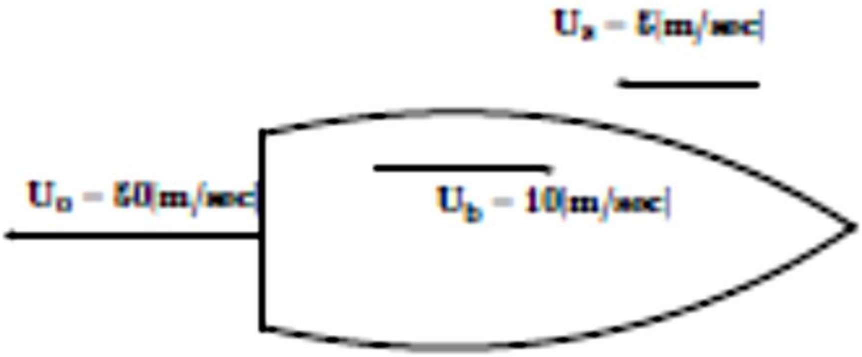
Thus, if all the valves are open the tank will be filled. The time to completely filled the tank is

$$\frac{\frac{1}{4}}{\frac{1}{2} - \frac{131}{280}} = \frac{70}{159} hour$$

Example 5.14:

*A boat travels at speed of 10*m/sec *upstream in a river that flows at a speed of 5*m/s. *The inboard engine uses a pump to suck*

in water at the front $A_{in} = 0.2\,m^2$ and eject it through the back of the boot with exist area of $A_{out} = 0.05\,m^2$. The water absolute velocity leaving the back is 50m/sec, what are the relative velocities entering and leaving the boat and the pumping rate?



<u>SOLUTION</u>

The boat is assumed (implicitly is stated) to be steady state and the density is constant. However, the calculation have to be made in the frame of reference moving with the boat. The relative jet discharge velocity is

$$U_{r_{out}} = 50 - (10 + 5) = 35[m/sec]$$

The volume flow rate is then

$$Q_{out} = A_{out}\,U_{r_{out}} = 35 \times 0.05 = 1.75 m^3/sec$$

The flow rate at entrance is the same as the exit thus,

$$U_{r_{in}} = \frac{A_{out}}{A_{in}}\,U_{r_{out}} = \frac{0.05}{0.2} 35 = 8.75 m/sec$$