Multiplication principle

If there are p experiments and the first has n_1 equally likely outcomes, the second has n_2 equally likely outcomes, and so on until the pth experiment has n_p equally likely outcomes, then there are

$n_1 \times n_2 \times .... \times n_p = \prod_{i=1}^{p} n_i$

equally likely possible outcomes for the p experiments.

Example

A class of school children consists of 14 boys and 17 girls. The teacher wishes to pick one boy and one girl to star in the school play.

By the multiplication principle, she can do this in 14×17 = 238 different ways.

The multiplication rule

The formula for conditional probability is useful when we want to calculate P $(A|B)$ from P $(A \cap B)$ and P (B). However, more commonly we want to know P $(A \cap B)$ and we know P $(A|B)$ and P (B).

A simple rearrangement gives us the multiplication rule.

P $(A \cap B)$ = P (B) × P $(A|B)$

The multiplication rule generalizes to more than two events.

For example, for three events we have,

P $(A_1 \cap A_2 \cap A_3)$ =P (A_1) P $(A_2 | A_1)$ P $(A_3|A_1 \cap A_2)$

Examples

1. What is the probability of getting an odd number when throwing dice

   P (1 or 3 or 5) =p (1) +p (3) +p (5)

   $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$ (these are mutually exclusive)

2. What is the probability of getting two faces are similar when rolling the dice twice

   P[(1,1) or (2,2)or (3,3) ….or(6,6)]=[p(1,1)+p(2,2)+p(3,3)+…+p(6,6)]

   $= \frac{1}{36} + \frac{1}{36} + \cdots .. + \frac{1}{36} = \frac{6}{36} = 1/6$ (these are mutually exclusive)

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Example:- Scoring Goals, If the probability of:

- **scoring no goals (Event "A") is 20%**
- **scoring exactly 1 goal (Event "B") is 15%**

Then:

- **The probability of scoring no goals and 1 goal is 0 (Impossible)**
- **The probability of scoring no goals or 1 goal is 20% + 15% = 35%**

Which is written: P (A ∩ B) = 0

P (A ∪ B) = 20% + 15% = 35%

Example:- A number is chosen at random from the set of two-digit numbers from 10 to 99 inclusive. What is the probability the number contains at least one digit 2?

If A is the event 'The units digit is 2', then A={12, 22, 32, 42, 52, 62, 72, 82, 92}
If B is the event 'The tens digit is 2',then B={20, 21, 22, 23, 24, 25, 26, 27, 28, 29}
A and B are not mutually exclusive since 22 is in both sets.

Use P(A or B)=P(A)+P(B)-P(A and B)

$$P(A) = \frac{9}{90},\ P(B) = \frac{10}{90} \text{ and } \mathrm{P(A\ and\ B)} = \frac{1}{90}$$

$$\text{So } \mathrm{P(A\ or\ B)} = \frac{9}{90} + \frac{10}{90} - \frac{1}{90} = \frac{18}{90} = \frac{1}{5}$$

Example: - Two fair dice are thrown.
What is the probability that the score on the first die is 6 or the score on the second die is 5?

These are not mutually exclusive events:
* First die is a 6: (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)
* Second die is a 5: (1,5) (2,5) (3,5) (4,5) (5,5) (6,5)
Notice they both contain (6, 5)

So we must use: P(A or B) = P(A) + P(B) - P(A and B)

$$\text{P(the score on the first die is 6)} = \frac{1}{6}$$

$$\text{P(the score on the second die is 5)} = \frac{1}{6}$$

$$\text{P}(6{,}5) = \frac{1}{36}$$

So the probability that the score on the first die is 6 or the score on the second die is 5 =

$$\frac{1}{6}+\frac{1}{6}-\frac{1}{36}=\frac{6}{36}+\frac{6}{36}-\frac{1}{36}=\frac{11}{36}$$

Independence
Two events A, B are independent if P (A, B) = P (A) P (B).
Equivalently, two events A and B are independent
If P (B | A) = P (B) or P (A|B) = P (A)

Remark

- If we can calculate P ($\mathbf{A}\cap \boldsymbol{B}$) directly, we can check the independence of A and B by seeing if it is true that P ($A\cap B$) = P (A).P (B)

- We can generalize independence to collections of events as follows. The set of events $A = \{A_1, A_2, \ldots, A_n\}$ are *mutually independent events* if *for any subset*, $B \subseteq A$, $B = \{B_1, B_2, \ldots, B_r\}$; $r \leq n$ we have
$P(B_1 \cap B_2 \cap \ldots \cap B_r) = P(B_1) \times \ldots \times P(B_r)$

Independence Rule
Using the formula for conditional probability in the definition of independence, we get that two events A and B are independent if
P(B) = P(B | A) =P(A ∩ B)/P(A)
P(A ∩ B) = P(A) ・ P(B)

Example:- (Flipping a coin two times)
The outcome of the second toss is in no way dependent on the outcome of the first toss. So the probability of landing two heads in a row is the

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probability of heads on the first toss times the probability of heads on the second toss=1/2 • 1/2=1/4.

Example:-(Flip a coin twice)
Another way to see this is to list the possible outcomes:

first toss	second toss
Heads	Heads
Heads	Tails
Tails	Heads
Tails	Tails

P (two heads) =1/4.

Example:-Auto Repairing shop on single occupancy, contains from 5 to 14 cars according to the following probabilities.

Numbers of car	5	6	7	8	9	10	11	12	13	14
Prob,	.05	0	.17	.08	.11	.23	.06	.14	.13	.03

Calculate the prob,

1. Reform less than 11 cars in one day job.
2. Reform more than 9 cars in one day job.
3. Reform from 8 to 13 cars in one day job.
4. Reform less than 4 cars in one day job.
5. Reform more than 20 cars in one day job.

S={5,6,….,14}

1. A={5,6,7,8,9,10}
   P(A)=P(5)+ P(6)+ P(7)+ ……+P(10)
   =.05+.00+.17+.08+.11+.23=64%
2. B={10,11,12,13,14}
   P(B)=P(10)+ P(11)+ P(12)+ P(13)+ P(14)
   =.23+.06+.14+.13+.03=59%
3. C={8,….,13}
   P(C)=p(8)+…..+p(13)
   =.08+.11+.23+.06+.14+.13=75%
4. D={ } ,p(D)=0
5. E={ },P(E)=0

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