Conservation of energy: Bernoulli's equation Consider the most important case of steady,

Several renewable energy resources derive from the natural movement of air and water. Therefore the transfer of energy to and from a moving fluid is the basis of meteorology and of hydro, wind, wave and some solar power systems. Examples of such applications include hydropower turbines (Figures 8.3, 8.5 and 8.6), wind turbines (picture on front cover and Figure 9.4), solar air heaters (Figure 6.1) and wave energy systems (Figure 12.14). Tounderstandsuchsystems,wemuststartwiththebasiclawsofmechanics as they apply to fluids, notably the laws of conservation of mass, energy and momentum. The term fluid includes both liquids and gases, which, unlike solids, do not remain in equilibrium when subjected to shearing forces. The hydrodynamic distinction between liquids and gases is that gases are easily compressed, whereas liquids have volumes varying only slightly with temperature and pressure. Gaseous volumes vary directly with temperature and inversely with pressure, approximately as the perfect-gas law $pV = nRT$. Nevertheless, for air, flowing at speeds $<100 ms^{-1}$ and not subject to large imposed variations in pressure or temperature, density change is negligible; this is the situation for the renewable energy systems analysed quantitatively in this book. It does not apply to the analysis of gas turbines, for which specialist texts should be consulted. Therefore, throughout this text, moving air is considered to have the fluid dynamics of an incompressible fluid. This considerably simplifies the analysis of most renewable energy systems.
Manyimportantfluidflowsarealsosteady,i.e.theparticulartypeofflow pattern at a location does not vary with time. So it is useful to picture a set of lines, called streamlines, parallel with the velocity vectors at each point. A further distinction is between laminar and turbulent flow (Section 2.5). For example, watch the smoke rising from a smouldering taper in still air. Near the taper, the smoke rises in an orderly, laminar, stream, with the paths of neighbouring smoke particles parallel. Further from the taper, the30 Essentials of fluid dynamics

flow becomes chaotic, turbulent, with individual smoke particles intermingling in three dimensions. Turbulent flow approximates to a steady mean flow, subject to internal friction caused by the velocity fluctuations. However, even in turbulence, the airflow remains within well-defined (though imaginary) streamtubes, as bounded by streamlines.

incompressible flow. At first, we assume no work is done by the moving fluid on, say, a hydro turbine.. The tube is narrow in comparison with other dimensions, so z is considered constant over each cross-section of the tube. A mass $m= A_1u_1 t$ enters the control volume at 1, and an equal mass $m= A_2u_2 t$ leaves at 2 (where is the density of the fluid, treated as constant). Then the energy balance on the fluid within the control volume is potential energy lost+work done by pressure forces =gain in kinetic energy+heat losses due to friction and may be written as

$mg\ z_1-z_2 + p_1A_1\ u_1\ t - p_2A_2\ u_2\ t = 1\ 2$

$m\ u_2\ 2-u_2\ 1 + E_f$ (2.1) where the pressure force p_1A_1 acts through a distance $u_1\ t$, and similarly for p_2A_2, and E_f is the heat generated by friction.

We neglect fluid friction, E_f, now, but we will examine some of its effects in Section 2.5. In this ideal, frictionless case, (2.1) reduces to

$p_1 + gz_1 + 1\ 2u_2\ 1 = p_2 + gz_2 + 1\ 2u_2\ 2$ (2.2) or, equivalently, so each term has the dimension of height (m)

p g +z+

u2 2g =constant along a streamline, with no loss of energy (2.3)

Either of these forms of the equation is called Bernoulli's equation. Equations (8.10) and (9.19) are examples of its application in hydro and wind power respectively. The sum of the terms on the left of (2.3) is called the total head of fluid (H). It relates to the total energy of a unit mass of fluid, however the constant in (2.3) may vary from streamline to streamline. Moreover, for many situations, the friction losses, Ef, have to be included. Head has the dimensions of length. For hydropower, head is the effective height of the moving water column incident on the turbine – see Section 8.3. The main limitation of (2.2) and (2.3) is that they apply only to fluids treated as ideal, i.e. with zero viscosity, zero compressibility and zero thermal conductivity. However, this is applicable to wind and hydro turbines with their relatively low-speed movement of air and water, and with no internal heat sources. The energy equation can however be modified to include non-ideal characteristics (see Bibliography), as for combustion engines and many other thermal devices, e.g. high temperature solar collectors. In solar heating systems and heat exchangers, power Pth is added to the fluid from heat sources (Figure 2.2). Heat E=Pth t is added to the energy inputs on the left hand side of (2.1). The mass m coming into the control volume at temperature T1 has heat content mcT1 (where c is the specific heat capacity of the fluid), and that going out has heat content mcT2. Thus we add to the right hand side of (2.1) the net heat carried out of the control volume in time t, namely mc T2−T1 . This gives an equation corresponding to (2.2), namely

p1 +gz1+

1 2

u2 1+cT1+

Pth Q = p2 +gz2+1 2

u2 2+cT2 (2.4)

where the volume flow rate is

Q=Au (2.5)

. Thermal power Pth is added to the flow.

In most heating systems, including active solar heating (see Equation 5.2 onwards), thermal contributions dominate the energy balance. So (2.4) reduces to Pth = cQ T2−T1 (2.6)

2.3 Conservation of momentum Newton'ssecondlawofmotionmaybedefinedforfluidsas:'Atanyinstant in steady flow, the resultant force acting on the moving fluid within a fixed volume of space equals the net rate of outflow of momentum from the closed surface bounding that volume.' This is known as the momentum theorem. As an example, consider a fluid passing across a turbine in a pipe. In Figure 2.3, fluid flowing at speed u1 into the left of the control surface carries momentum u1 per unit volume in the direction of flow. In time t, the volume entering the surface is A1u1 t. Therefore the rate at which the momentum is entering the control surface along the x direction is

A1u1 t u1 t = A2u2 1 (2.7) Similarly the rate at which momentum is leaving the control volume is A1u2 2. The momentum theorem tells us that the rate of change of momentum equals the force, F on the fluid and the reaction,−F is the force exerted on the turbine and pipe by the fluid. So F = A2u2 2−A1u2 1 =˙ mu2−˙ mu1 (2.8)