Introduction

A digital logic system may well have a numerical computation capability as well as its inherent logical capability and consequently it must be able to implement the four basic arithmetic processes of addition, subtraction, multiplication and division.
Human beings normally perform arithmetic operations using the decimal number system, but, by comparison, a digital machine is inherently binary in nature and its numerical calculations are executed using a binary number system.
Since the decimal system has ten digits, a ten-state device is required to represent the decimal digits, one state being allocated to each of the decimal digits.
Ten-state devices are not readily available in the electrical world, however two-state devices such as a transistor operating in a switching mode are, and it is forth is reason that the binary number system is of great importance to the digital engineer. In addition to the binary system, a number of other systems such as the hexadecimal system are used in.

Number Systems

A number system defines how a number can be represented using distinct symbols. A number can be represented differently in different systems. For example, the two numbers $(2A)_{16}$ and $(52)_8$ both refer to the same quantity, $(42)_{10}$, but their representations are different.

The decimal system (base 10)

The word decimal is derived from the Latin root decem (ten). In this system the base b = 10 and we use ten symbols

$$\mathbf{S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}}$$

The symbols in this system are often referred to as decimal digits or just digits.

Integers

$$N = \pm \quad S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \ldots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

	10^{k-1}	10^{k-2}	• • •	10^2	10^1	10^0	Place values
$\pm$	S_{k-1}	S_{k-2}	• • •	S_2	S_1	S_0	Number
$N = \pm$	$S_{k-1} \times 10^{k-1}$ +	$S_{k-2} \times 10^{k-2}$ +	• • • +	$S_2 \times 10^2$ +	$S_1 \times 10^1$ +	$S_0 \times 10^0$	Values

Figure 2.1 Place values for an integer in the decimal system

EX.1

The following shows the place values for the integer +224 in the decimal system.

	10^2		10^1		10^0	Place values
	2		2		4	Number
$N = +$	2×10^2	+	2×10^1	+	4×10^0	Values

Note that the digit 2 in position 1 has the value 20, but the same digit in position 2 has the value 200. Also note that we normally drop the plus sign, but it is implicit

EX.2

The following shows the place values for the decimal number −7508. We have used 1, 10, 100, and 1000 instead of powers of 10.

	1000		100		10		1	Place values
	7		5		0		8	Number
$N = -($	7×1000	+	5×100	+	0×10	+	8×1	) **Values**

Note that the digit 2 in position 1 has the value 20, but the same digit in position 2 has the value 200. Also note that we normally drop the plus sign, but it is implicit.

Reals

	Integral part		Fractional part
$R = \pm$	$S_{k-1} \times 10^{k-1} + \ldots + S_1 \times 10^1 + S_0 \times 10^0$	+	$S_{-1} \times 10^{-1} + \ldots + S_{-l} \times 10^{-l}$

The following shows the place values for the real number +24.13.

EX.3

	10^1		10^0		10^{-1}		10^{-2}	Place values
	2		4	•	1		3	Number
$R = +$	2×10	+	4×1	+	1×0.1	+	3×0.01	Values

The binary system (base 2)

The word binary is derived from the Latin root bini (or two by two). In this system the base b = 2 and we use only two symbols,

S = {0, 1}

The symbols in this system are often referred to as binary digits or bits (binary digit).

$$N = \pm\ S_{k-1} \times 2^{k-1} + S_{k-2} \times 2^{k-2} + \ldots + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0$$

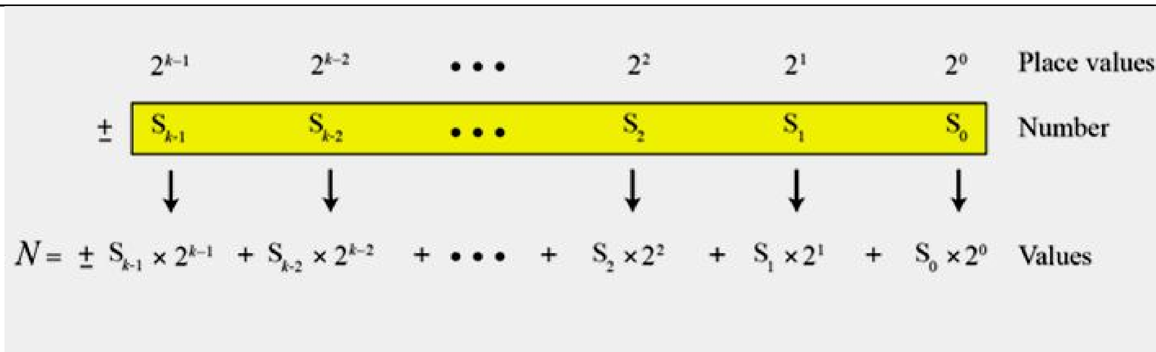

Figure 2.2 Place values for an integer in the binary system

EX.4

The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2.

		2^4		2^3		2^2		2^1		2^0	Place values
		1		1		0		0		1	Number
N	=	1×2^4	+	1×2^3	+	0×2^2	+	0×2^1	+	1×2^0	Decimal

The equivalent decimal number is N = 16 + 8 + 0 + 0 + 1 = 25.

Reals

$$R = \pm \underbrace{S_{k-1} \times 2^{k-1} + \ldots + S_1 \times 2^1 + S_0 \times 2^0}_{\text{Integral part}} \; \bullet \; + \underbrace{S_{-1} \times 2^{-1} + \ldots + S_{-l} \times 2^{-l}}_{\text{Fractional part}}$$

EX.5

The following shows that the number $(101.11)_2$ in binary is equal to the number 5.75 in decimal.

		2^2		2^1		2^0		2^{-1}		2^{-2}		Place values
		1		0		1	•	1		1		Number
R	=	1×2^2	+	0×2^1	+	1×2^0	+	1×2^{-1}	+	1×2^{-2}		Values

The octal system (base 8)

The word octal is derived from the Latin root **octo** (eight). In this system the **base b = 8** and we use eight symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

Integers

$$N = \pm S_{k-1} \times 8^{k-1} + S_{k-2} \times 8^{k-2} + \ldots + S_2 \times 8^2 + S_1 \times 8^1 + S_0 \times 8^0$$

	8^{k-1}	8^{k-2}	•••	8^2	8^1	8^0	Place values
$\pm$	S_{k-1}	S_{k-2}	•••	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	
$N =$	$\pm S_{k-1} \times 8^{k-1}$	$+ S_{k-2} \times 8^{k-2}$	+ ••• +	$S_2 \times 8^2$	$+ S_1 \times 8^1$	$+ S_0 \times 8^0$	Values

Figure 2.3 Place values for an integer in the octal system

EX.6

The following shows that the number $(1256)_8$ in octal is the same as 686 in decimal.

		8^3		8^2		8^1		8^0		Place values
		1		2		5		6		Number
N	=	1×8^3	+	2×8^2	+	5×8^1	+	6×8^0		Values

Note that the decimal number is N = 512 + 128 + 40 + 6 = 686.

The hexadecimal system (base 16)

The word **hexadecimal** is derived from the Greek root **hex** (six) and the Latin root **decem** (ten). In this system the **base b = 16** and we use sixteen symbols to represent a number. The set of symbols is

S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

Note that the symbols A, B, C, D, E, F are equivalent to 10, 11, 12, 13, 14, and 15 respectively. The symbols in this system are often referred to as **hexadecimal digits**.

Integers

$$N = \pm S_{k-1} \times 16^{k-1} + S_{k-2} \times 16^{k-2} + \ldots + S_2 \times 16^2 + S_1 \times 16^1 + S_0 \times 16^0$$

	16^{k-1}	16^{k-2}	•••	16^2	16^1	16^0	Place values
$\pm$	S_{k-1}	S_{k-2}	•••	S_2	S_1	S_0	Number
$N = \pm$	$S_{k-1} \times 16^{k-1}$ +	$S_{k-2} \times 16^{k-2}$ +	••• +	$S_2 \times 16^2$ +	$S_1 \times 16^1$ +	$S_0 \times 16^0$	Values

Figure 2.3 Place values for an integer in the hexadecimal system

EX.6

The following shows that the number $(2AE)_{16}$ in hexadecimal is equivalent to 686 in decimal.

	16^2		16^1		16^0	Place values
	2		A		E	Number
N =	2×16^2	+	10×16^1	+	14×16^0	Values

The equivalent decimal number is N = 512 + 160 + 14 = 686.

End of the lecture