Steady-State Conduction— One Dimension

HEAT TRANSFER in COMPOSITE WALL :-

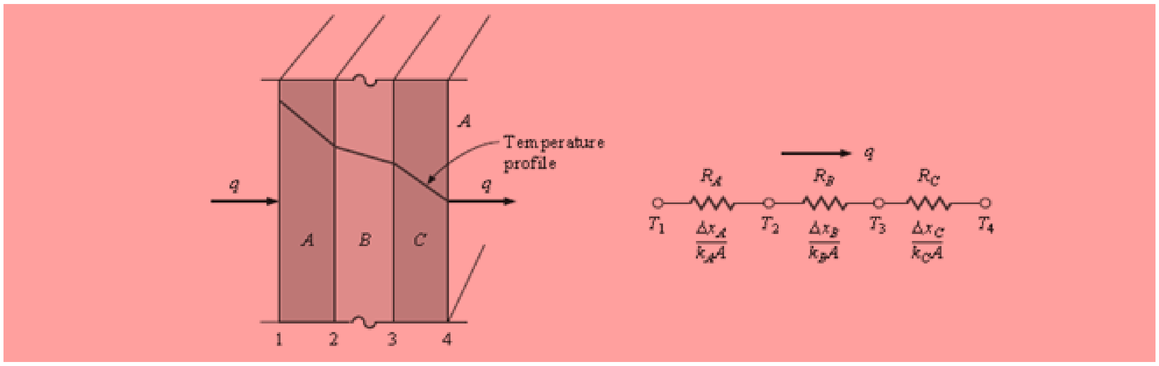


Fig3. One –dimensional heat transfer through a composite wall and electrical analog .

$$R_A = \frac{\Delta X_A}{K_A A} \qquad R_B = \frac{\Delta X_B}{K_B A} \qquad R_C = \frac{\Delta X_C}{K_C A}$$

$$q_A = -K_A\, A\, \frac{T_2 - T_1}{\Delta X_A} \qquad q_B = -K_B\, A\, \frac{T_3 - T_2}{\Delta X_B} \qquad q_C = -K_C\, A\, \frac{T_4 - T_3}{\Delta X_C}$$

$$q = K_A\, A\, \frac{T_1 - T_2}{\Delta X_A} = K_B\, A\, \frac{T_2 - T_3}{\Delta X_B} = K_C\, A\, \frac{T_3 - T_4}{\Delta X_C}$$

$$q = \frac{T_1 - T_4}{\dfrac{\Delta X_A}{K_A A} + \dfrac{\Delta X_B}{K_B A} + \dfrac{\Delta X_C}{K_C A}}$$

$$\text{Heat flow} = \frac{\text{thermal potential difference}}{\text{thermal resistance}}$$

$$q = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{th}}}$$

where the R_{th} are the thermal resistances of the various materials. The units for the thermal resistance are °C/W or °F · h/Btu.

Example 1:

A wall 2 cm thick is to be constructed from material that has an average thermal conductivity of 1.3 W/m.◦C. The wall is to be insulated with material having an average thermal conductivity of 0.35W/m.◦C, so that the heat loss per square meter will not exceed 1830 W/m². *Assuming that the inner and outer surface temperatures of the insulated wall are 1300 and 30◦C, calculate the thickness of insulation required.*

Solution:

$$q = \frac{T_i - T_o}{\frac{\Delta X_A}{kA})_w + \frac{\Delta X_B}{kA})_{ins}}$$

$$1830 = \frac{1300 - 30}{\frac{0.02}{1.3} + \frac{\Delta X}{0.35}}$$

$$\Delta X = 0.238 \text{ m}$$

INSULATION AND *R* VALUES

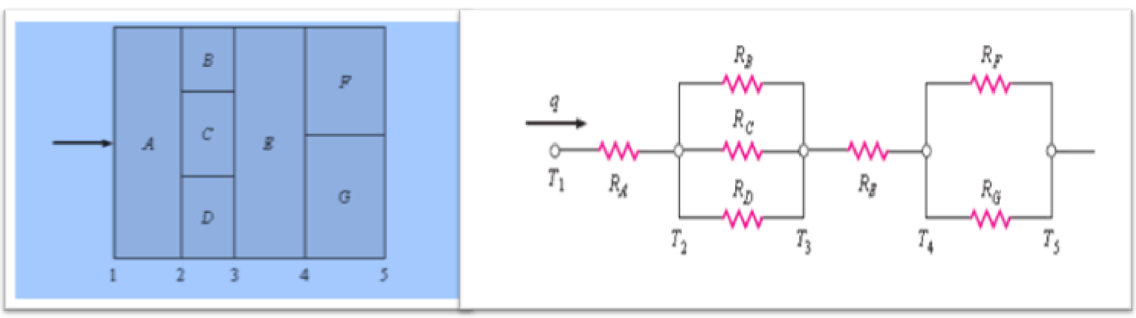


Fig4. *Series and parallel one- dimensional heat transfer through a composite wall and electrical analog .*

In the building industry to use a term called the R value, which is defined as $R = \frac{\Delta T}{q/A}$

The units for *R* are $°C.\frac{m}{W}$ ***or*** $°F.ft^2/$**Btu.**

$$q_A = k\,A\,\frac{T_1 - T_2}{\Delta X_A}$$

$$q_B, q_C, q_D = K\,A\,\frac{T_2 - T_3}{\Delta X_B}, K\,A\,\frac{T_2 - T_3}{\Delta X_C}, K\,A\,\frac{T_2 - T_3}{\Delta X_D}$$

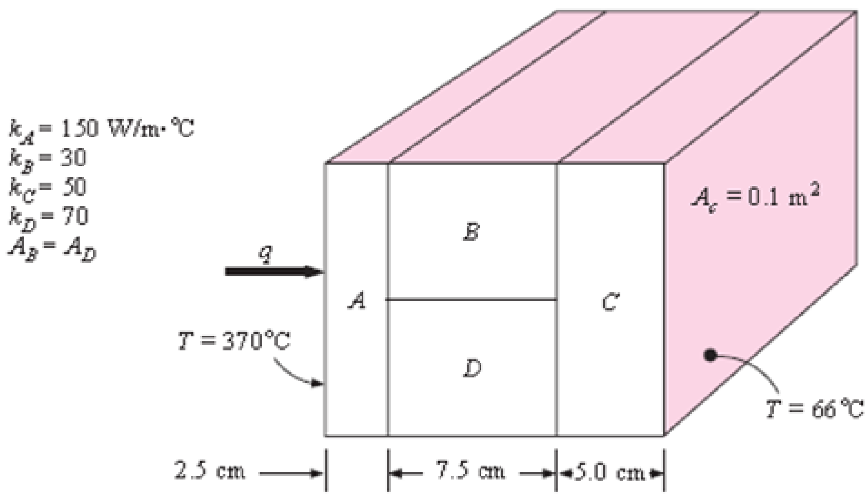
$$q_E = K\,A\,\frac{T_3 - T_4}{\Delta X_E}$$

$$q_F, q_G = K\,A\,\frac{T_4 - T_5}{\Delta X_F}, K\,A\,\frac{T_4 - T_5}{\Delta X_G}$$

Example :

Find the heat transfer per unit area through the composite wall in Figure P2-4. Assume one-dimensional heat flow.

Figure P2-4



Solution:

$$R = \frac{\Delta X}{KA}$$

$$R_A = \frac{0.025}{(150)(0.1)} = 1.667*10^{-3}$$

$$R_B = \frac{0.075}{(30)(0.05)} = 0.05$$

$$R_C = \frac{0.05}{(50)(0.1)} = 0.01$$

$$R_D = \frac{0.075}{(70)(0.05)} = 0.02143$$

$$R = R_A + R_C + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} = 2.667*10^{-2}$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{370-66}{2.667*10^{-2}} = 11400 \text{ w}$$

RADIAL SYSTEMS

➢ Cylinders:-

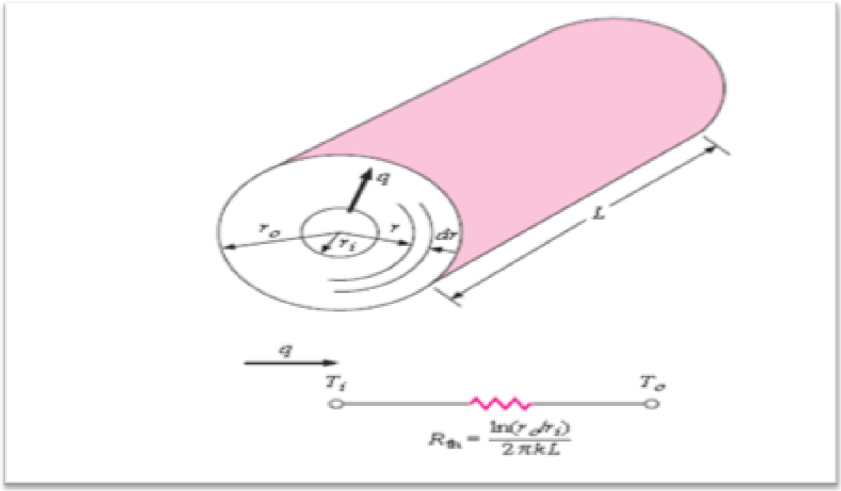


Fig5. One –dimensional heat flow through a hollow cylinder and electrical analog.

$A_r = 2\pi r L$

so that **Fourier's law** is written:

$$q_r = -kA\frac{dT}{dr}$$

$$q_r = -k * 2\pi r L\ \frac{dT}{dr}$$

with the boundary conditions:-

$T = T_i$ at $r = r_i$

$T = T_o$ at $r = r_o$

$$q.\frac{dr}{r} = -K\ 2\pi L\ .dT$$

$$q.\int_{r_i}^{r_o}\frac{dr}{r} = -K\ 2\pi L\ \int_{T_i}^{T_o} dT$$

$$q.\ln\frac{r_o}{r_i} = -K\ 2\pi L(T_o - T_i)$$

$$q = \frac{-K\ 2\pi L(T_o - T_i)}{ln\frac{r_o}{r_i}}$$

$$\boxed{\therefore q = \frac{2\pi kL(T_i - T_o)}{ln\frac{r_o}{r_i}}}$$

Example :

A steel tube having k =46W/m °C has an inside diameter of 3.0 cm and a tube wall thickness of 2 mm. A fluid flows on the inside of the tube producing a convection coefficient of 1500 W/m² °C on the inside surface, while a second fluid flows across the outside of the tube producing a convection coefficient of 197 W/m² °Con the outside tube surface. The inside fluid temperature is 223 °C while the outside fluid temperature is 57 °C. Calculate the heat lost by the tube per meter of length.

Solution:

$A = \pi d l$
مساحة سطحية حيث الطول واحد.

$$R_{convection} = \frac{1}{h_i A_i} = \frac{1}{1500\pi(0.03)} = 0.00709 \ °C/w$$

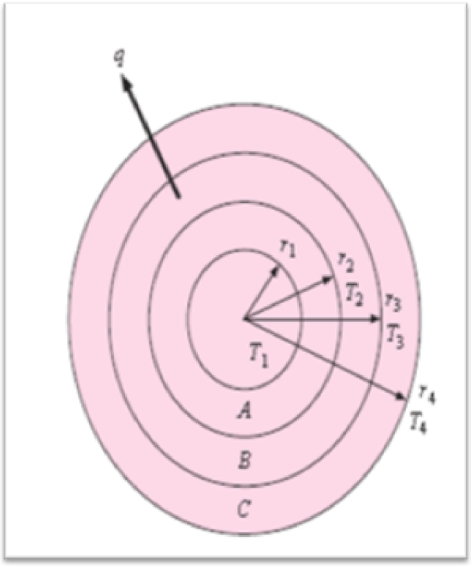
$$R_{cylinder} = \frac{ln(\frac{r_o}{r_i})}{2\pi LK} = \frac{\ln(\frac{0.017}{0.015})}{2\pi(46)} = 0.000433 \ °C/w$$

$r_o = r_i + thickness$
= 0.015+0.002
= 0.017

$$R_{convection} = \frac{1}{h_o A_o} = \frac{1}{(197)\pi(0.034)} = 0.0475 \ °C/w$$

$$\Sigma R = 0.05502 \ °C/w$$

$$q/L = \frac{\Delta T}{\Sigma R} = \frac{223-57}{0.05502} = 3017 \ w/m$$



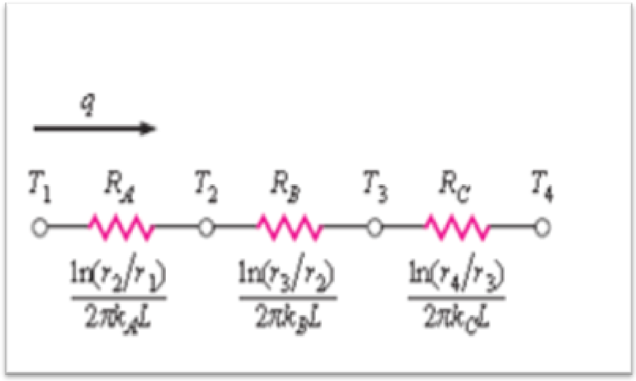


*Fig6. One –dimensional heat flow through **multiple cylindrical** section and electrical analog.*

$$q= \frac{2\pi L(T_1-T_4)}{\ln(\frac{r2}{r1})/K_A+\ln(\frac{r3}{r2})/K_B+\ln(\frac{r4}{r3})/K_C}$$

RADIAL SYSTEMS

➢ Sphere :-

$$q= -kA_r\frac{dT}{dr}$$

$$= -k(4\pi r^2)\frac{dT}{dr}$$

$$q.\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$$

$$q.-\frac{1}{r}\Big]_{r_1}^{r_2} = -4\pi K\ (T_2-T_1)$$

$$q= \frac{4\pi k(T_1-T_2)}{\frac{1}{r_1}-\frac{1}{r_2}}$$

Example :

A hollow sphere is constructed of aluminum with an inner diameter of 4 cm and an outer diameter of 8 cm. The inside temperature is 100◦C and the outer temperature is 50◦C. Calculate the heat transfer.(K=204w/m.◦C)

Solution:

$$q=\frac{4\pi k(T_i-T_o)}{\frac{1}{r_i}-\frac{1}{r_o}}$$

$$q=\frac{4\pi(204)(100-50)}{\frac{1}{0.02}-\frac{1}{0.04}}=5127\ \text{w}$$

Notes :

Conduction Resistance (plane wall) $\boxed{R_{wall}=\frac{L}{KA}}$

Conduction Resistance (cylinder) $\boxed{R_{cylinder}=\frac{\ln(\frac{r_2}{r_1})}{2\pi LK}}$

Conduction Resistance (sphere) $\boxed{R_{sphere}=\frac{r_2-r_1}{4\pi K r_1 r_2}}$

Convection Resistance $\boxed{R_{convection}=\frac{1}{hA}}$

Radiation Resistance $\boxed{R_{convection}=\frac{1}{h_{rad}A}}$

$$h_{rad}=\varepsilon\sigma(T_s^2-T_{surr}^2)(T_s-T_{surr})$$