6-10-2001

1 مراجع

Fluid III
6,10,01

Sat 6-14

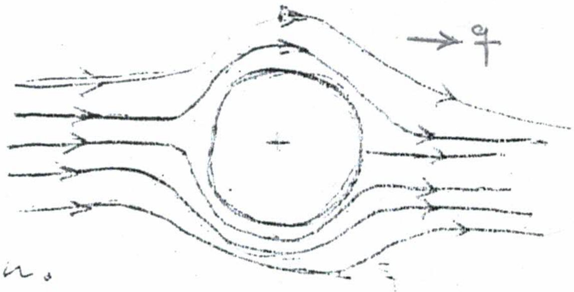
Chapter One

* Ideal Fluid Flow :-

شعاع السرعة

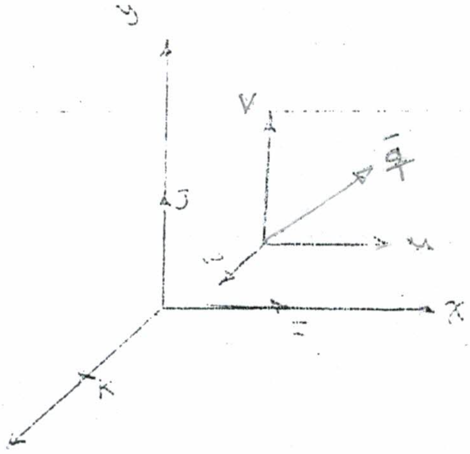
$\bar{q}$ = Velocity Vector

The objective is to determine the Flow around a Solid body .. i.e. to Find the Velocity and thus the pressure distribution.



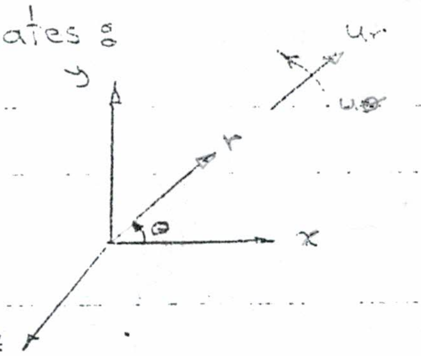
* In Cartesian coordinates :-

$$\vec{q} = u\vec{i} + v\vec{j} + w\vec{k}$$



In Cylinderical (polar) Coordinates :

$$\bar{q} = u_r \bar{r} + u_\theta \bar{\theta} + w\bar{k}$$



(2)

∇ = Gradient (or nabla operator)

$$\nabla = \bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}$$

انتبه $\nabla\cdot$ = divergence.

$\nabla \times q$ = Curl of $\vec{q}$.

$\nabla\cdot\vec{q}$ = divergence of $\vec{q}$ ($div\ \vec{q}$)

or

$$\nabla\cdot\vec{q} = \left[\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}\right]\cdot\left[u\bar{i} + v\bar{j} + w\bar{k}\right]$$

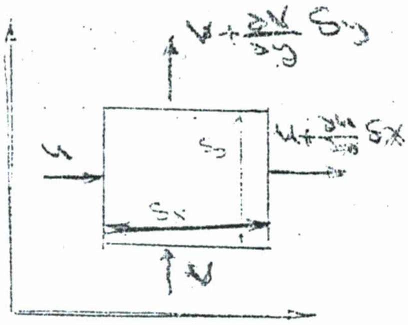
Then:-

$$\boxed{\nabla\cdot\vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}} \text{ — (2)}$$ معادلة الاستمرارية

Equation (2) $\equiv$ Continuity equation. معادلة الاستمرارية

Proof:-

consider the element $\delta x\,\delta y$ in an incompressible, steady, two-dimensional flow.



in flow = out flow

$$u\,\delta y + v\,\delta x = \left(u + \frac{\partial u}{\partial x}\delta x\right)\delta y + \left(v + \frac{\partial v}{\partial y}\delta y\right)\delta x$$

$$\Rightarrow \cancel{u\delta y} + \cancel{v\delta x} = \cancel{u\delta y} + \frac{\partial u}{\partial x}\delta x\delta y + \cancel{v\delta x} + \frac{\partial v}{\partial y}\delta x\delta y$$

$$\frac{\partial u}{\partial x}\delta x\delta y + \frac{\partial v}{\partial y}\delta y\delta x = 0 \rightarrow \text{divided by } \delta x\delta y$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$ ← continuity equ. for 2-D Flow.

and in 3D Flow.

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

معادلة الاستمرارية

$\therefore \nabla\cdot\vec{q} \equiv$ Continuity equation

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انتبه $\nabla \times = \text{Curl}$

$\nabla \times \vec{q} = \text{Curl of } \vec{q}$

or

$$\nabla \times \vec{q} = \left[\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{K}\frac{\partial}{\partial z}\right] \overset{\text{Cross}}{\times} \left[u\bar{i} + v\bar{j} + w\bar{K}\right]$$

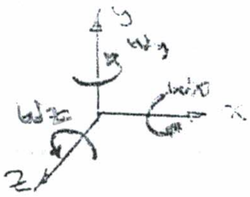
$$= \begin{vmatrix} +\bar{i} & -\bar{j} & +\bar{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \text{ ---- then}$$

معادلة الدوامات

$$\boxed{\nabla \times \vec{q} = \bar{i}\underbrace{\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)}_{w_x} + \bar{j}\underbrace{\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)}_{w_y} + \bar{K}\underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}_{w_z}} \quad \text{---- (3)}$$

Equation (3) $\equiv$ Vorticity equation معادلة الدوامات..

$\nabla \times \vec{q} = \bar{i} w_x + \bar{j} w_y + \bar{K} w_z$



* For two dimensional Flow

$\boxed{w_x = 0}$ & $\boxed{w_y = 0}$. $\boxed{w_z \neq 0}$

if $\nabla \times \vec{q} \neq 0$ at every point in a Flow, the Flow is called «rotational».

if $\nabla \times \vec{q} = 0$ at every point in a Flow, the Flow is called «irrotational».

(4)

1.2 Requirements For ideal-Fluid Flow :-

(1)- non Viscous. ($\mu = zero$).

(2)- in compressible. ($\rho = Const$). $\frac{\delta\rho}{\delta t} = 0$, $\frac{\partial\rho}{\delta x} = 0$, $\frac{\delta\rho}{\delta y} = 0$, $\frac{\delta\rho}{\delta z} = 0$

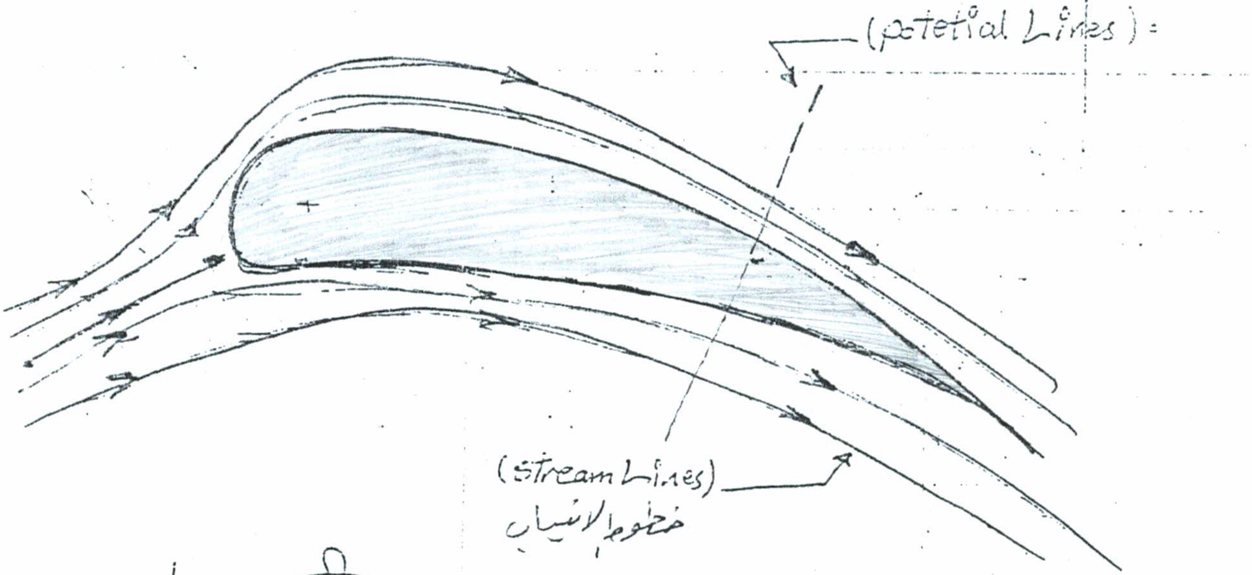
(3)- the continuity equation $\Rightarrow$ $\boxed{\nabla \cdot q = 0}$. $\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\right]$

(4)- irrotational Flow $\boxed{\nabla \times \vec{q} = 0}$. $[i\omega_x + j\omega_y + k\omega_z = 0]$

* In Order to determine the Flow around a Solid body we shall define the Following.

دالة الانسياب

* (1)- Stream Function. (which define the Stream Lines.)
* (2)- potential Function. (which define the potential Lines.)



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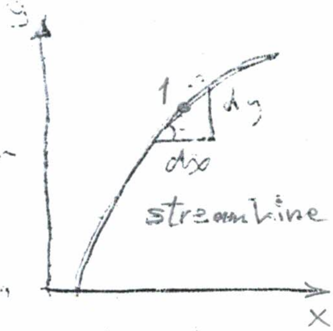
1.3 Stream Function :- دالة الانسياب (ψ)

the Slope of the Stream Line at point(1)

is:- $\tan\theta = \dfrac{v}{u} = \dfrac{dy}{dx}$



$\therefore$ $\boxed{\dfrac{dy}{dx} = \dfrac{v}{u}}$ — (*) differential Equation For a Stream Line.

if u and v Known Function of x's and y then eqn (*) can be intergrated to yield the algebraic equation for a stream Line. $F(x,y) = Const$

The Function $F(x,y)$ is called the Stream Function and is denoted by the $\boxed{\text{Symbol } \psi}$

$\therefore$ $\boxed{\psi(x,y) = Const}$ → equation for a streamLine.

مركبات السرعة

The relation ship between the Velocity components and the Stream Function ψ is:

مركبات السرعة

$$\boxed{u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}} \longrightarrow (4a)$$

and

$$\boxed{u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad u_\theta = -\frac{\partial \psi}{\partial r}} \longrightarrow (4b)$$

Summary الخلاصة

(6)- ** Continuity equ in therms of ψ :-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

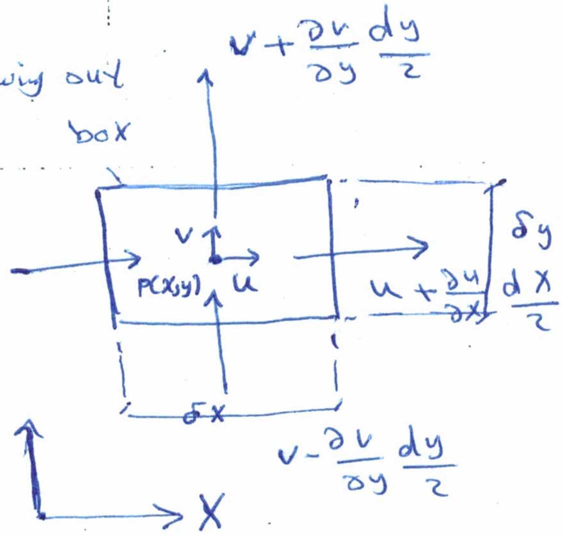
$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right)=0$$

$$\boxed{\frac{\partial^2 \psi}{\partial x \partial y}-\frac{\partial^2 \psi}{\partial y \partial x}=0}$$

* معادلة الاستمرارية بدلالة دالة الانسياب.

Continuity equation:

Volume/sec. flowing into = Volume/sec. flowing out box



$$\left(u-\frac{\partial u}{\partial x}\frac{\delta x}{2}\right)\delta y+\left(v-\frac{\partial v}{\partial y}\frac{\delta y}{2}\right)\delta x=$$

$$\left(u+\frac{\partial u}{\partial x}\frac{\delta x}{2}\right)\delta y+\left(v+\frac{\partial v}{\partial y}\frac{\delta y}{2}\right)\delta x$$

$$\therefore \frac{\partial u}{\partial x}\delta x\, \delta y+\frac{\partial v}{\partial y}\delta x\, \delta y=0 \quad \div \delta x \delta y$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$ (This equation is applicable to both viscous and inviscid fluids)

20-10-2001

* 1.4 Potential function or velocity potential (ϕ):-

For an irrotational flow:

$$\nabla \times \vec{q} = 0$$

the velocity potential ϕ may be defined as:-

$$\vec{q} = \nabla \phi$$

thus:-

$$\boxed{u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y}} \longrightarrow (6a)$$

In the Cylindrical Coordinates:

$$\boxed{u_r = \frac{\partial \phi}{\partial r} \qquad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}} \longrightarrow (6b)$$

** Continuity equation in terms of ϕ

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0} \longrightarrow (7)$$

Laplace eq. معادلة لابلاس

معادلة الاستمرارية بدلالة دالة الجهد

$$\nabla^2 \phi = 0$$

** Because irrotational flow can be described by the velocity potential "ϕ" irrotational flow is called potential flow

الجريان الجهدي.

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* Summery : الخلاصة

$$\boxed{\begin{aligned} u &= \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} \end{aligned}} \Longrightarrow (8) \text{ حفظ}$$

$$\boxed{\begin{aligned} u_r &= \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r} \\ u_\theta &= -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \end{aligned}}$$

Exs:-

1- Compute the gradient of the following 2-D scalar function

(a) $\phi = -2\ln(x^2+y^2)$

(b) $\phi = ux + vy$

(c) $\phi = 2xy$

Solution:- gradient $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + \cancel{k\frac{\partial}{\partial z}}^{\,0}$

(a) $\nabla\phi = \nabla(-2\ln(x^2+y^2))$

$$\nabla\phi = \frac{-4x}{x^2+y^2}\, i - \frac{4y}{x^2+y^2}\, j$$

(b) $\nabla\phi = \nabla(ux+vy) \Rightarrow ui + vj$

(c) $\nabla\phi = \frac{\partial \phi}{\partial x}\, \overset{\circ}{i} + \frac{\partial \phi}{\partial y}\, \overset{\circ}{j} + \cancel{\frac{\partial \phi}{\partial z}}^{\,0} k$

$$\nabla\phi = 2y\,\bar{i} + 2x\,\overset{\circ}{j}$$

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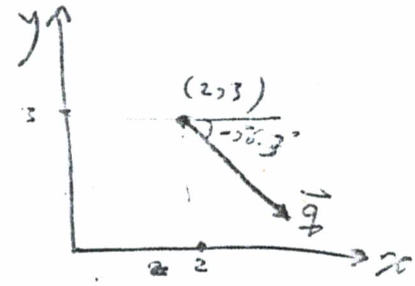
Ex 2:- Does the stream function $\psi = xy$ represent a physically possible flow? If so determine the velocity at a point $(2,3)$

Sol:- the Continuity eq. must be satisfied

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

$1 - 1 = 0$ (إذنه يوجد جريان)



* لكي نحسب سرعة النقطة (2,3) يجب معرفة مركبات السرعة.

$$u = \frac{\partial \psi}{\partial y} = x$$

$$v = -\frac{\partial \psi}{\partial x} = -y$$

مركبات السرعة

at point $(2,3)$ $\quad u = 2, \quad v = -3$

$\vec{q} = \sqrt{u^2 + v^2} = \underline{3.6}$ units.

$\theta = \tan^{-1} \frac{v}{u} = -56.3°$

Ex3:- Which of the following function could represent the velocity potential for the 2-D flow of an ideal fluid

a- $x + 5y$

b- $3x^2 + 4y^2$

c- $\cos(x - y)$

d- $\tan^{-1}\left(\frac{x}{y}\right)$

$\cos x \cos y - \sin x \sin y$

$= -\sin x \cos y - \cos x \sin y$

$= -\cos x \cos y + \sin x \sin y$

$-\cos x \sin y - \sin x \cos y$

$-\cos x \cos y + \sin x \sin y$

Sol:- Continuity eq.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

(a) $\phi = x + 5y \Rightarrow 0 + 0 = 0$ ✓

(b) $6 + 8 \neq 0$

(c)

(d)

4. **Ex 4**:- A velocity potential in 2-D fluid are given by (a) $\phi = y + x^2 - y^2$

(b) $\phi = 2\sin x + 2\cos^2 y + 6$

Find the stream function for these flows..

Sol:.. (a) $\phi = y + x^2 - y^2$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 2x$$

$$\partial \psi = 2x \cdot \partial y \Rightarrow d\psi = 2x\,dy$$

$$\psi = 2xy + f(x) \text{ ——— } *$$ where $f(x)$ = constant of integration.

To find $f(x)$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$

$$-(2y + f'(x)) = 1 - 2y \Rightarrow 2y + f'(x) = -1 + 2y$$

$$\therefore f'(x) = -1 \Rightarrow \int f'(x)\,dx = -x + c$$

Sub in eq (*)

$$\boxed{\psi = 2xy - x + c} \text{ Ans}$$

b) $\phi = 2\sin x + 2\cos^2 y + 6$ $\Rightarrow$ Ans; $\psi = 4x\cos y \sin y + c$

$$u = \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = 2\cos x$$

$$\psi = 2y\cos x + f(x)$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}$$

$2(1 - \sin 2y)$

$$+2y\sin x + f'(x) = -4\cos y \sin y$$

$$f(x) = -4x\cos y \sin y - 2y\cos x + c$$

Ex5 :- A stream function in 2-D flow :-

$$\psi = 9 + 6x - 4y + 7xy$$

find ϕ.

Sol :- $u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = -4 + 7x$

$\partial \phi = (-4 + 7x)\,\partial x \Rightarrow \phi = \int (-4 + 7x)\,dx$

$\therefore \phi = -4x + \frac{7}{2}x^2 + f(y)$

To find $f(y)$;

$V = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

$f'(y) = -(6 + 7y) = -6 - 7y$

$\therefore f(y) = -6y - \frac{7}{2}y^2 + c$

$\therefore$ ~~[illegible]~~ $\therefore \boxed{\phi = -4x - 6y + \frac{7}{2}(x^2 - y^2) + c}$ Ans

---

Ex6 :- $u = \frac{y^3}{3} + 2x - x^2 y$

$V = xy^2 - 2y - \frac{x^3}{3}$

[a] Show that these function represent a possible case of irrotational flow

[b] obtain an expressions for ψ & ϕ

Sol :- [a] $\nabla \cdot \vec{q} = 0$ continuity eq $= \checkmark$

$\nabla \times \vec{q} = 0$ irrotational $= \checkmark$

* the continuity eq. and the condition of irrotational flow must be satisfied ;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \overset{?}{=} 0 \Rightarrow (2 - 2xy) + (2xy - 2) = 0$$

b) for two dia (2-D) flow

$\therefore \omega_x = 0$, $\omega_y = 0$, $\omega_z \overset{?}{=} 0$

$$\omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \overset{?}{=} 0$$

$$(y^2 - x^2) - (y^2 - x^2) = 0$$

$\therefore \omega_z = 0$ $\therefore$ the flow is irrotational.

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[b] $u = \dfrac{\partial \psi}{\partial y}$

$$\therefore \psi = \int u \, dy = \frac{y^4}{12} + 2xy - \frac{x^2 y^2}{2} + f(x)$$

* <u>To find $f(x)$</u>

$$v = -\frac{\partial \psi}{\partial x}$$

$$\cancel{xy^2} - \cancel{2y} - \frac{x^3}{3} = -\cancel{2y} + \cancel{xy^2} - f'(x)$$

$$f'(x) = \frac{x^3}{3}$$

$$\therefore f(x) = \frac{x^4}{12} + c$$

$$\therefore \psi = \frac{y^4}{12} + 2xy - \frac{x^2 y^2}{2} + \frac{x^4}{12} + c$$

<u>Ans for ϕ</u>

$$\boxed{\phi = \frac{xy^3}{3} + x^2 - \frac{x^3 y}{3} - y^2 + c}$$

$$u = \frac{\partial \phi}{\partial x} \Rightarrow \partial \phi = \partial x \, u = d\phi = u \, dx = \int d\phi = \int \frac{y^3}{3} + 2x - x^2 y \, dx$$

$$\phi = \frac{y^3 x}{3} + x^2 - \frac{x^3 y}{3} + \underline{\underline{f(y)}}$$

To find $f(y)$:-

$$v = \frac{\partial \phi}{\partial y} = \cancel{xy^2} - 2y - \cancel{\frac{x^3}{3}} = \cancel{y^2 x} - \cancel{\frac{x^3}{3}} + f'(y) \Rightarrow \int f'(y) = \int -2y \, dy$$

$$f(y) = -y^2 + c \Rightarrow \phi = \frac{xy^3}{3} + x^2 - \frac{x^3 y}{3} - y^2 + c$$

7// **Ex 7**:- $u = 2xy$, $v = x^2 - y^2$

تحقق حفظ

Show that the flow is irrotational and satisfies Conservation of

الكتلة

mass.

Sol:- $\boxed{\nabla \cdot \vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$ ← حفظ الكتلة = معادلة الاستمرارية

$2y - 2y + 0 = \boxed{0 + 0 = 0}$

$\therefore \boxed{\nabla \cdot \vec{q} = 0}$ متحققة

$\nabla \times \vec{q} = \omega_x i + \omega_y j + \omega_z k$

* For two diamensional flow $\omega_x = 0$, $\omega_y = 0$

$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 2x - 2x = 0$

$\therefore \boxed{\nabla \times \vec{q} = 0}$ $\therefore$ irrotational flow.

Ex 8:- A flow is defined by $u = 2x$ and $v = -2y$

check Continuity and check to see if the flow is irrotational.

Find the stream function and potential function. Draw the stream lines. Corresponding to $\psi = 100, 300$

also potential lines Corresponding to $\phi = -300, -100, 0, 100, 300$

Solution:- Continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \overset{?}{=} 0 \Rightarrow 2 - 2 = 0$

for 2-D $\omega_x = 0$ $\omega_y = 0$

$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \overset{?}{=} 0 \Rightarrow 0 - 0 = 0$

$\therefore$ flow is irrotational $u = \frac{\partial \psi}{\partial y}$

$\psi = \int u\, dy = 2xy + f(x)$

also: $v = -\frac{\partial \psi}{\partial x}$.

$-2y = -2y - f'(x) \Rightarrow f'(x) = 0$

$\therefore f(x) = C$

$\therefore \boxed{\psi = 2xy + C}$

* ψ = ابسايل
* ϕ = ثاي

$u = \frac{\partial \phi}{\partial x} \Rightarrow \phi = \int u\,dx = x^2 + f(y)$

also $v = \frac{\partial \phi}{\partial y} \Rightarrow -2y = f'(y)$

$\therefore f(y) = -y^2 + C \Rightarrow \therefore \boxed{\phi = x^2 - y^2 + C}$

for $C = 0$ (C may be any value = constant) اي قيمة ممكن تكون اي قيمة نختارها

$\boxed{\psi = 2xy}$

$\psi = 100 = 2xy$

x:	0	1	2	5	10	25	∞
y:	∞	50	25	10	5	2	0

$\psi = 300 = 2xy$

x:	0	5	10	15	20	25	30	∞
y:	∞	30	15	10	7.5	6	5	0

$\phi = -300 = x^2 - y^2$

x:	0	5	10	20	25	30
y:	17.3	18	20	26.5	30.4	34.6

$\phi = -100 = x^2 - y^2$

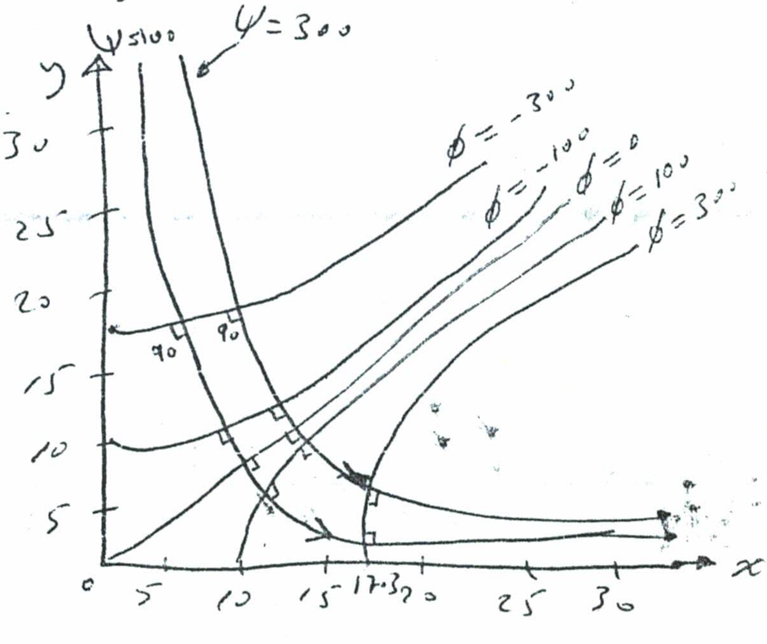
x:

y:

$\phi = 0 = x^2 - y^2$

x:

y:



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$\underline{\phi = 100 = x^2 - y^2}$

x : 10 , 11 , 12 , 15 , 20 , 25

y : 0 , 4.59 , 6.64 , 11.2 , 17.3 , 22.9.

$\underline{\phi = 300 = x^2 - y^2}$

x :

y :

* تكون تقاطع بين خطوط الانسياب
وخطوط الجهد دائماً قائمة (90°).