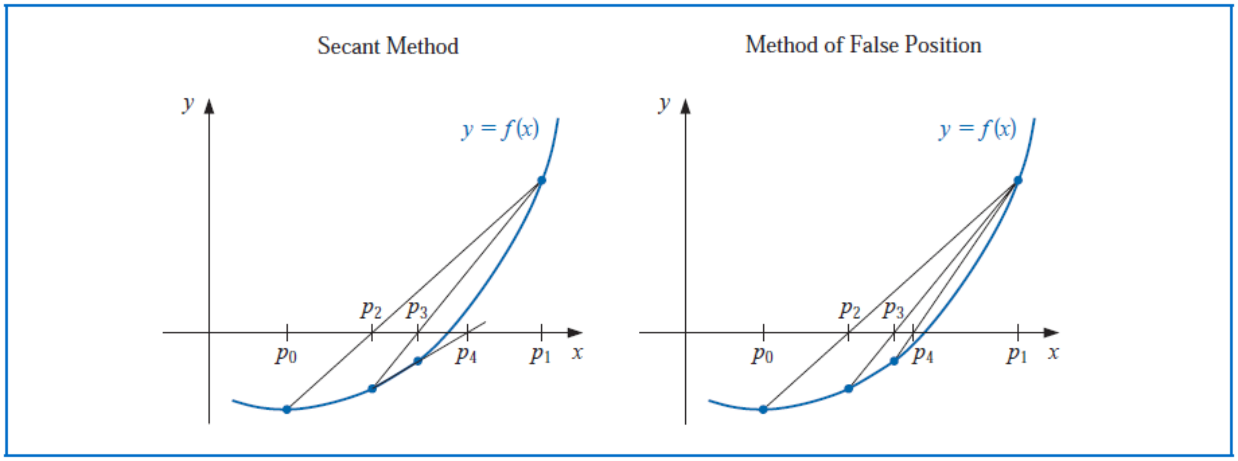
False Position Method

The **method of False Position** generates approximations in the same manner as the Secant method, but it includes a test to ensure that the root is always bracketed between successive iterations.

First choose initial approximations p_0 and p_1 with $f(p_0) \cdot f(p_1) < 0$. The approximation p_2 is chosen in the same manner as in the Secant method, as the x-intercept of the line joining $(p_0, f(p_0))$ and $(p_1, f(p_1))$. To decide which secant line to use to compute p_3, consider $f(p_2) \cdot f(p_1)$;

- If $f(p_2) \cdot f(p_1) < 0$, then p_1 and p_2 bracket a root. Choose p_3 as the x-intercept of the line joining $(p_1, f(p_1))$ and $(p_2, f(p_2))$.
- If not, choose p_3 as the x-intercept of the line joining $(p_0, f(p_0))$ and $(p_2, f(p_2))$.
- In a similar manner compute p_4, and so on.



Example1: Find the root of xln(x)-1=0 using False Position in the interval [1, 2] and ε=0.001

Solution:

$f(x_1)=-1$, $f(x_2)=0.3863$

$$x = \frac{1 * (0.3863) - 2 * (-1)}{0.3863 + 1} = 1.7213$$

$|x_1 - x| = |1 - 1.7213| = 0.7213 > \varepsilon$

$f(x) = -0.0652$

$x_1 = x$; $y_1 = y$;

$$x = \frac{1.7213 * (0.3863) - 2 * (-0.0652)}{0.3863 - (-0.0652)} = 1.7615$$

$|x_1 - x| = |1.7213 - 1.7615| = 0.0402 > \varepsilon$

$f(x) = -0.0027$

$x_1 = x$; $y_1 = y$;

$$x = \frac{1.7615 * 0.3863 - 2 * (-0.0027)}{0.3863 - (-0.0027)} = 1.7632$$

$|x_1 - x| = |1.7615 - 1.7632| = 0.0017 > \varepsilon$

$f(x) = -0.00004$

$x_2 = x$; $y_2 = y$;

Example2:

Use the method of False Position to find a solution to $x = \cos x$. Compare the results with fixed point, Newton and Secant methods.

Solution: To make a reasonable comparison we will use the same initial approximations as in the Secant method, that is, $p_0 = 0.5$ and $p_1 = \pi/4$. In the table below, notice that the False Position and Secant approximations agree through p_3 and that the method of False Position requires an additional iteration to obtain the same accuracy as the Secant method.

	False Position	Secant	Newton
n	p_n	p_n	p_n
0	0.5	0.5	0.7853981635
1	0.7853981635	0.7853981635	0.7395361337
2	0.7363841388	0.7363841388	0.7390851781
3	0.7390581392	0.7390581392	0.7390851332
4	0.7390848638	0.7390851493	0.7390851332
5	0.7390851305	0.7390851332	
6	0.7390851332		

Exercises:

Q Use Secant, Newton and false position methods to find solutions to within 10^{-7} for the following problems.

a) $x^2 - 4x + 4 - \ln x = 0$ for $1 \le x \le 2$ and for $2 \le x \le 4$

b) $x + 1 - 2 \sin \pi x = 0$ for $0 \le x \le 1/2$ and for $1/2 \le x \le 1$