Secant Method

By definition

$$f'(p_{n-1}) = \lim_{x \to p_n} \frac{f(x) - f(p_{n-1})}{x - p_{n-1}}$$

If p_{n-2} is close to p_{n-1}, then

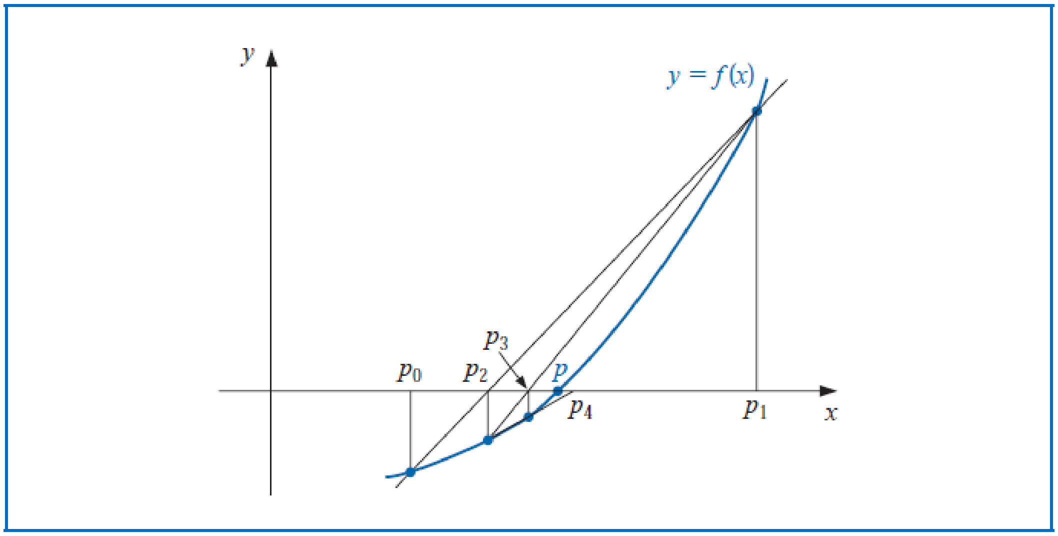
$$f'(p_{n-1}) \approx \frac{f(p_{n-2}) - f(p_{n-1})}{p_{n-2} - p_{n-1}} = \frac{f(p_{n-1}) - f(p_{n-2})}{p_{n-1} - p_{n-2}}$$

Using the approximation of $f'(p_{n-1})$ in Newton's formula gives

$$p_n = p_{n-1} - \frac{f \quad (p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

This technique is called the **Secant method.** Starting with the two initial approximations p_0 and p_1, the approximation p_2 is the x-intercept of the line joining $(p_0, f(p_0))$ and $(p_1, f(p_1))$. The approximation p_3 is the x-intercept of the line joining $(p_1, f(p_1))$ and $(p_2, f(p_2))$, and so on. If $x_1=p_1$, $x_2=p_2$, $y_1=f(x_1)$ and $y_2=f(p_2)$, then

$$x = x_2 - \frac{y_2(x_2 - x_1)}{y_2 - y_1} = \frac{x_1 . y_2 - x_2 . y_1}{y_2 - y_1}$$



Example:

Find the root of $f(x) = x\ln(x) - 1$ using Secant Method in the interval [1,2],and ε =0.0018.

Solution:

x_1=1; x_2=2; f(x_1)=-1; f(x_2)=0.3863;

|2-1|=1>ε

$$x = \frac{1 * 0.3863 - 2 * (-1)}{0.3863 - (-1)} = 1.7213$$

f(x)=-0.0652

x_1=x_2=2; y_1=y_2=0.3863; x_2=x=1.7213; y_2=y=-0.0652;

|1.7213 – 2|=0.2787>ε

$$x = \frac{2 * (-0.0652) - 1.7213 * 0.3863}{-0.0652 - 0.3863} = 1.7615$$

f(x)= -0.0027

x_1=x_2=1.7213; y_1=y_2= -0.0652; x_2=x= 1.7615; y_2=y= -0.0027;

|1.7615 – 1.7213|=0.0402>ε

$$x = \frac{1.7213 * (-0.0027) - 1.7615 * (-0.0652)}{-0.0027 - (-0.0652)} = 1.7632$$

f(x)= -3.5784e-5;

x_1=x_2=1.7615; y_1=y_2= -0.0027; x_2=x=1.7632; y_2=y= -3.5784e-5;

|1.7632 - 1.715|=0.0017<ε;

Then the root is x= 1.7632

Exercises:

Q\ The fourth-degree polynomial f (x) = $230x^4 + 18x^3 + 9x^2 - 221x - 9$ has two real zeros, one in [−1, 0] and the other in [0, 1]. Attempt to approximate these zeros to within 10^{-6} using the

a. Method of False Position

b. Secant method

c. Newton’s method

Use the endpoints of each interval as the initial approximations in (a) and (b) and the midpoints as the initial approximation in (c).