Biostatistics

Lecture 5

Probability

The concept of probability is not a foreign to health workers.

-For example we may hear a physician say that a patient has 50-50 chance surviving from a certain operation.

- Or physician may say that a patient has 95% a particular disease.

What is the probability?

Probability is a measure (or number) used to measure the chance of the occurrence of some event, this number is between 0 and 1.

Probability may be Classified into:

1- Classical probability.

2- Relative frequency of probability.

1- Classical probability

There are many conceptions in Classical probability such as:

- Probability: is a measure (or number) used to measure the chance of the occurrence of some event. This number is between 0 and 1.

- Sample Space: The set of all possible outcomes of an experiment is called the sample space (or Universal set) and is denoted by Ω .

- **Mutually exclusive:** is defined as the occurrence of one event exclude the occurrence of any others, or two event are said to be mutually exclusive if they can not occur simultaneously.
- **Equally likely:** is the outcomes of an experiment are equally likely if the occurrences of the outcomes have the same chance.
- **Probability of an event:** If the experiment has (n) equally likely outcomes, then the probability of the event (E) is:

$$P(E) = \frac{n(E)}{n(\Omega)} = \frac{n(E)}{N} = \frac{no.\ of\ outcomes\ \ in\ \ E}{no.\ of\ outcomes\ \ in\ \ \Omega}$$

Example: **If a card is drawn from ordinary deck, find the probability that is a heart. The number of possible outcome is 52, of which 13 hearts.**

P (E) = 13/52 = 14

2- Relative frequency of probability

The relative frequency of an event (E).

$$P(E) = \frac{fi}{\sum fi}$$

<u>Example:</u> If we toss a coin 100 time and find it comes up head 60 times. We estimate the probability of head to be,

P (H) = 60/100 = 0.6

$\cup$

Some Operations on Events

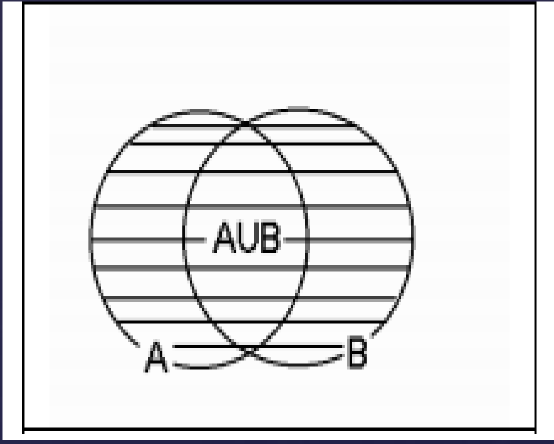
1- Ungrouped data

Example: Let A and B be two events defined on:

A= {Patients 1, 2, 3, 4, 5, 6} =all assigned patients who are receiving drug therapy.

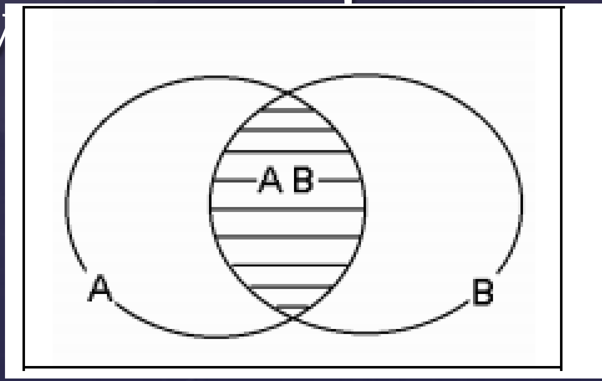
B= {Patients 2, 4, 7, 8, 9, 10, 11} =all assigned patients who are receiving psychotherapy.

1. The **Union** events ($\cup$) consists of all elements belonging to either A or B or both A and B, if A occurs, or B occurs, or both A and B occur, from above example.



$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

2. The Intersection events ($\cap$): of two sets, A and B, is another set and consist of all outcomes elements that are in both A and B. if both A and B occur, from above example:



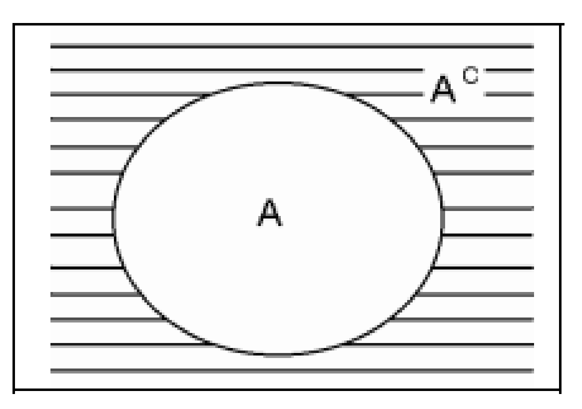
= Equal all assigned patients receiving both drug therapy and psychotherapy.

$A \cap B$ = {2, 4}

A= {Patients 1, 2, 3, 4, 5, 6} =all assigned patients who are receiving drug therapy.

B= {Patients 2, 4, 7, 8, 9, 10, 11} =all assigned patients who are receiving psychotherapy.

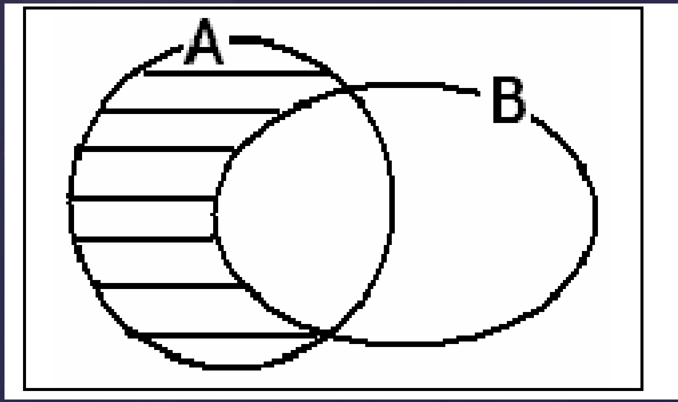
3. The **Complement** events ($\overline{A}$ = not): consists of all outcomes of Ω but are not in A.



$$\overline{A} = (7,8,9,10,11)$$

A= {Patients 1, 2, 3, 4, 5, 6} =all assigned patients who are receiving drug therapy.
B= {Patients 2, 4, 7, 8, 9, 10, 11} =all assigned patients who are receiving psychotherapy.

4. **The Difference (A - B):** The set consisting of all elements of (A) which do not belong to (B) is called the difference A and B.



A – B = {1, 3, 5, 6}

A= {Patients 1, 2, 3, 4, 5, 6} =all assigned patients who are receiving drug therapy.

B= {Patients 2, 4, 7, 8, 9, 10, 11} =all assigned patients who are receiving psychotherapy.

Factorial (!): Given the positive integer n, is the product of all the whole numbers from n down through 1.

$$n! = n(n-1)\ (n-2)\ \ldots\ 1$$

Example: 5!

$$= 5(5-1)(5-2)(5-3)(5-4)$$

$$=5 \times 4 \times 3 \times 2 \times 1 = 120$$

Note: 0! = 1 by definition.

Permutation is an ordered arrangement of objects.

1- Permutation of <u>different</u> elements for objects.

Example: Considering the four letters a, b, c, d the permutations or arrangements of these letter two at each time is 12 numbers.

ab	ac	ad	bc	bd
ca	cd	da	db	dc

$$ {}_nP_r = \frac{n!}{(n-r)!} \qquad {}_4P_2 = \frac{4!}{(4-2)!} = \frac{4\times 3\times 2\times 1}{2\times 1} = 12 $$

Permutation:

2- Permutation of <u>same</u> elements for objects.

Permutation are different if they contain the same letters but in a different order. If the objects to be arranged are same and there are p of the first kind, q of the second kind, r of the third kind and s for the fourth, the number of permutations of these objects is,

$$= \frac{n!}{p! \times q! \times r! \times s!}$$

Where n = p + q + r + s

Example: How many permutations of letter (BIOSTATICS)?

B = 1 I = 3 O =1 S =3 T =3

A =1 C =1

n = 1+3+1+3+3+1+1 =13

$$\frac{13!}{1!\times 3!\times 1!\times 3!\times 3!\times 1!\times 1!} = 28828800$$

Combinationis an arrangement of objects without regard to order

$$\binom{n}{r}C = \frac{n!}{r!(n-r)!}$$

The number of combinations of the four letters a, b, c, d taken two at each
ac ad bc bd cd time is 6 numbers: ab

$$\binom{4}{2}C = \frac{4!}{2!(4-2)!} = 6$$

Example: Four lunches' patients in a hospital may choose one of four meats, two of five vegetable, and one of three desserts. How many combinations of different meals does the patient have from which to choose if he selects the specified number from each group?

$$\binom{4}{1} = \frac{4!}{1!(4-1)!} = 4 \quad \text{..................... (Meats)}$$

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = 10 \quad \text{.................... (Vegetables)}$$

$$\binom{3}{1} = \frac{3!}{1!(3-1)!} = 3 \quad \text{...................... (Desserts)}$$

$$\binom{4}{1} \times \binom{5}{2} \times \binom{3}{1} = 4 \times 10 \times 3 = 120$$

1- If A and B are any events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2- If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B)$$

Some Probability Laws

because $$P(A \cap B) = 0$$

3- If A and B are complementary events, then

$$P(\overline{A}) = 1 - P(A)$$

4- The conditional probability of A, given B, then

$$P(A \setminus B) = \frac{p(A \cap B)}{P(B)}$$

$$P(B) > 0$$

5- The events A and B are <u>independent</u>, then $$P(A \cap B) = P(A) \times P(B)$$

6- The event A is <u>not affected</u> by the occurrence of event B, then

$$P(A \setminus B) = \frac{p(A \cap B)}{P(B)} = P(A)$$

Example: The following table shows 900 adults in a small city who have completed the requirements for a college degree. The results categorize according to sex and employment status:

	Employed (E)	Unemployed $(\overline{E})$	Total
Male (M)	460	40	500
Female $(\overline{M})$	140	260	400
Total	600	300	900

	Employed (E)	Unemployed $(\overline{E})$	Total
Male (M)	460	40	500
Female $(\overline{M})$	140	260	400
Total	600	300	900

$$1 - If\,(M\ and\,E)\,any\,events ------------------ P(M \cup E) = \frac{500}{900} + \frac{600}{900} - \frac{460}{900} = 0.7111$$

$$2 - If\,(\overline{M})\,complementary\,events -------------- P(\overline{M}) = 1 - \frac{500}{900} = 0.4444$$

$$3 - If\,(M\ given\,\overline{E})\,envents ------------------ P(M \cap \overline{E}) = \frac{40/900}{300/900} = 0.1332$$

$$4 - If\,(M\ and\,\overline{M})\,mutually\,exclusive\,events -------- P(M \cup \overline{M}) = \frac{500}{900} + \frac{400}{900} = 1.0000$$

$$5 - If\,(M)\,not\,effected\,(E)\,events -------------- P(M \setminus E) = \frac{460/900}{600/900} = 0.7667$$

$$6 - If\,(M\ and\,E)\,independent\,events ------------ P(M \cap E) = \frac{500}{900} \times \frac{600}{900} = 0.37$$