University of Babylon
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Lecture 2

Contents

1. Free Vibration of Single-Degree of- Freedom Systems

Free Vibration occurs when the system oscillates on its own due to initial disturbance without any external forces. For instant a simple pendulum oscillation and the motion of a swing after an initial disturbance.

A system shown in Figure.1 is a single-degree-of-freedom system, since one coordinate (x) is sufficient to specify the position of the mass at any time.

$F = K(x + l_0)$

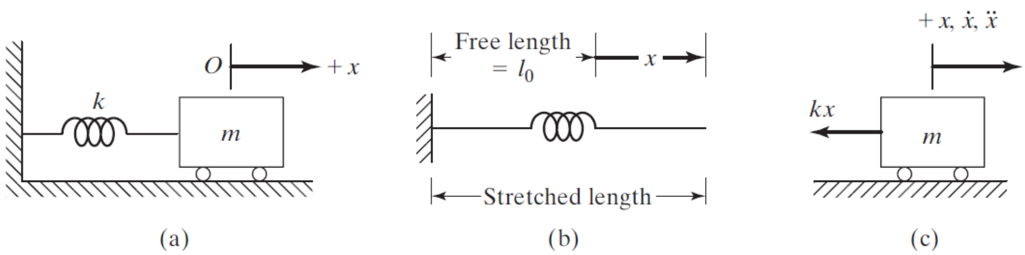


Figure1. A spring-mass system in horizontal position

2. Free Vibration of an Undamped Translational System

As the motion for the system in Figure.1 is linear therefore we can use Newton's second law to derive the equation of motion as follow;

$$\sum F = m\ddot{x}$$

According to the free body diagram in Figure.1-(c)

$$-kx = m\ddot{x}$$

$$m\ddot{x} + kx = 0$$

The solution of Equation of motion can be found as follow;

$$x(t) = Ce^{st}$$

$$\dot{x}(t) = Cse^{st}$$

$$\ddot{x}(t) = Cs^2e^{st}$$

$$m\,Cs^2e^{st} + kCe^{st} = 0$$

$$m\,s^2 + k = 0$$

Where C, s are constants,

$$s = \pm\sqrt{-\frac{k}{m}} = \pm i\omega_n$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$x(t) = C_1e^{i\omega_n} + C_2e^{-i\omega_n}$$

where C_1 and C_2 are constants. By using the identities

$$e^{\pm i\alpha t} = \cos\alpha t \pm \boldsymbol{i}\sin\alpha t$$

$$x(t) = A_1\cos\omega_n t + A_2\sin\omega_n t$$

where A_1 and A_2 are constants.

The constants C_1 and C_2 and A_1 and A_2 can be determined from the initial conditions of the system. Two conditions are to be specified to evaluate these constants uniquely. Note that the number of conditions to be specified is the same as the order of the governing differential equation.

At $t = 0$

$$x(t=0) = A_1 = x_0$$

$$\dot{x}(t=0) = \omega_n A_2 = \dot{x}_0$$

$$x(t) = x_0\cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t$$

2.1 For vertical vibration

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$W = mg = k\delta_{st}$$

$$k = \frac{W}{\delta_{st}} = \frac{mg}{\delta_{st}}$$

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}}$$

$$\omega_n = 2\pi f_n$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}$$

$$\tau_n = \frac{1}{f_n} = 2\pi\sqrt{\frac{\delta_{st}}{g}}$$

Thus, when the mass vibrates in a vertical direction, we can compute the natural frequency and the period of vibration by simply measuring the static deflection δ_{st} .We don't need to know the spring stiffness *k* and the mass ***m.***

Deflection in vertical direction can be calculated as follow;

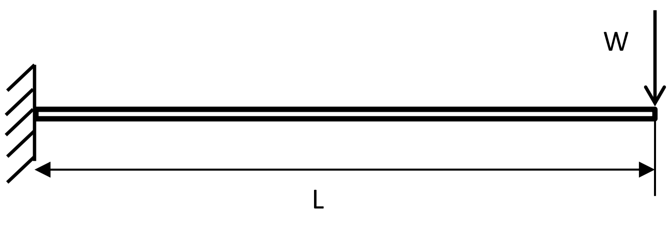
$$\delta = \frac{WL}{AE}$$

2.2 For transverse vibration

Deflection in transverse direction can be calculated as follow;

a) For Cantilever beam

$$\delta = \frac{WL^3}{3EI}$$



I= moment of inertia,

E=Young's Modulus,

L= beam length,

W=weight,

Moment of inertia

1) for circular section;

$$I = \frac{\pi d^4}{64}$$

2) for triangular section;

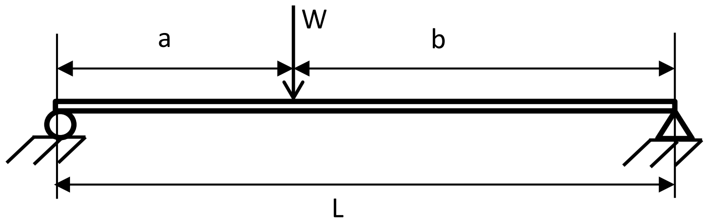
$$I = \frac{bh^3}{36}$$

3) for rectangular section;

$$I = \frac{bh^3}{12}$$

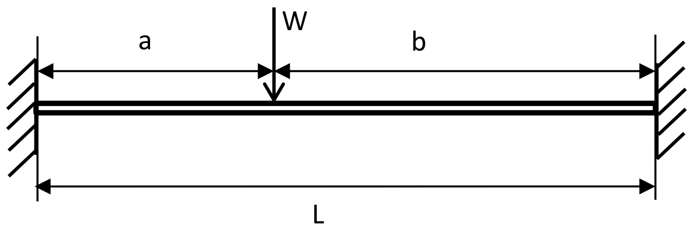
b) For Simply supported beam

$$\delta = \frac{Wa^2b^2}{3EIL}$$



c) For fixed beam

$$\delta = \frac{Wa^3b^3}{3EIL^3}$$



3. Free Vibration of an Undamped Torsional System

Torsional vibration occurs when a rigid body oscillates about a specific reference axis. Displacement of the body is measured in terms of an angular coordinate.

$$M_t = \frac{GI_0\theta}{l}$$

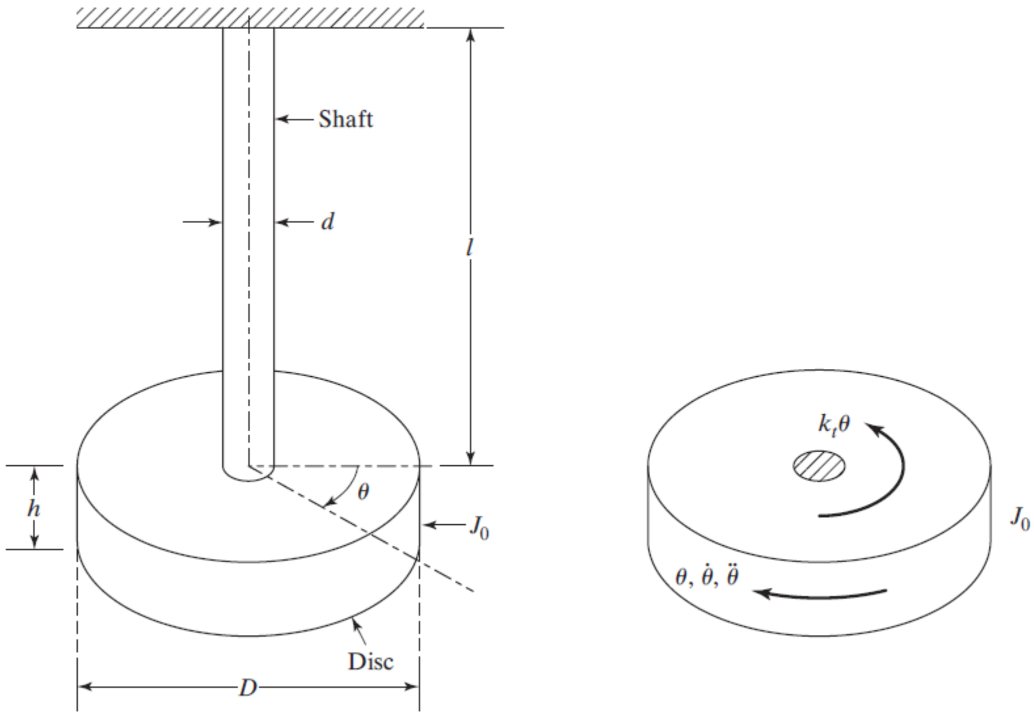
where M_t is the torque that produces the twist θ, G is the shear modulus, l is the length of the shaft, I_0 is the polar moment of inertia of the cross section of the shaft, given by

$$I_0 = \frac{\pi d^4}{32}$$

where the units for I_0 is (m^4)

If the disc is displaced by θ from its equilibrium position, the shaft provides a restoring torque of magnitude M_t. Thus the shaft acts as a torsional spring with a torsional spring constant

$$K_t = \frac{M_t}{\theta} = \frac{GI_0}{l}$$



The equation of the angular motion of the disc about its axis can be derived by using Newton's second law;

$$J_0\ddot{\theta} + k_t\theta = 0$$

where J_0 is the polar mass moment of inertia, θ is the angular displacement and k_t is the torsional spring constant.

$$\omega_n = \sqrt{\frac{k_t}{J_0}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_t}{J_0}}$$

$$\tau_n = \frac{1}{f_n} = 2\pi\sqrt{\frac{J_0}{k_t}}$$

The solution for the torsional vibration equation of motion is as follow;

$$\theta(t) = A_1 \cos \omega_n t + A_2 \sin \omega_n t$$

$A_1\ and\ A_2$ can be determined from the initial conditions;

At $t = 0$

$$\theta(t = 0) = A_1 = \theta_0$$

$$\dot{\theta}(t = 0) = \omega_n A_2 = \dot{\theta}_0$$

$$\theta(t) = \theta_0 \cos \omega_n t + \frac{\dot{\theta}_0}{\omega_n} \sin \omega_n t$$

where J_0 for a cylinder with radius r and mass m can be calculated as follow;

$$J_0 = \frac{1}{2}.m.r^2$$

and the units for J_0 is $(kg.m^2)$