Now , to find $\int_a^b f(x)\,dx$ $\quad \ldots \quad$ (6)

Where this integration is not determined by (-1,1)

First , we must change the variable of integration , we make the limits of integration from -1 to 1 by using the following form :

$$x = \frac{(b-a)t+(b+a)}{2} \quad \ldots \quad (7)$$

Hence : $dx = \left(\frac{b-a}{2}\right) dt$

The integration (6) becomes :-

$$\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f\big(x(t)\big)dt$$

i.e. $\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{(b-a)t+(b+a)}{2}\right) dt$

Example:

Find $\int_0^2 e^{x^2}\ dx$ by using G.L. where $n = 1$

Solution :

We must change the limits of integration :-

$$\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f\big(x(t)\big)dt$$

We know that $\int_{-1}^{1} f(x)dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$ before finding $\int_0^2 e^{x^2}\ dx$ we use the relation

$$x = \frac{(b-a)t + (b+a)}{2}$$

$$= \frac{2t+2}{2} \Rightarrow x = t+1$$

$$dx = dt$$

So , $\int_0^2 e^{x^2}\ dx = \int_{-1}^{1} e^{(t+1)^2} dt$

$$= e^{\left(-\frac{1}{\sqrt{3}}+1\right)^2} + e^{\left(\frac{1}{\sqrt{3}}+1\right)^2}$$

$\simeq$ 1.9558 + 12.2332 =13.2332

ملاحظة : -

يمكن مقارنة صيغة كاوس – لاجندر (n = 1) بطريقة شبه المنحرف (n = 1) فنجد ان الاولى هي اكثر دقة .

H.W. find $\int_0^2 x^3 \; dx$ by using

1. Trapezoidal , n=1
2. Gauss – Legendre , n=1

n=2

Gauss – Legendre methods

This method is called : Three- term Gaussian Legendre method .

We can find $I = \int_a^b f(x)dx$, n=2 by using

$$I = \int_a^b f(x)dx = \sum_{i=0}^{2} a_i \; f(x_i)$$

$$= a_0 \; f(x_0) + a_1 f(x_1) + a_2 f(x_2)$$

Example : - find $\int_{-1}^{1} e^x \; dx$, n=2 for four digit

Solution :

$$\int_{-1}^{1} f(x)dx = \; a_0 \; f(x_0) + a_1 f(x_1) + a_2 f(x_2)$$

$\therefore \int_{-1}^{1} e^x \; dx$ = 0.55555($e^{0.7746}$) +0.88888(e^0) + 0.55555($e^{0.7746}$)

$\simeq$ 3.2996

Example:

Use three –terms Gaussian Legendre Method to find $\int_0^1 e^{-x^2} \; dx$, for three decimal .

$$x = \frac{(b-a)t + (b+a)}{2}$$

$$= \frac{t+1}{2} \Rightarrow dx = \frac{1}{2}\ dt$$

$$\int_0^1 e^{-x^2}\ dx\ = \frac{1}{2} \int_{-1}^{1} e^{-\left(\frac{t+1}{2}\right)^2}\ dt$$

$$= \frac{1}{2}\ [\ 0.555 \left(e^{-\left(\frac{0.774+1}{2}\right)^2} \right) + 0.888 e^{\left(-\left(\frac{1}{2}\right)^2\right)} + 0.555 \left(e^{-\left(\frac{0.774+1}{2}\right)^2} \right)$$

$$= 0.5975$$