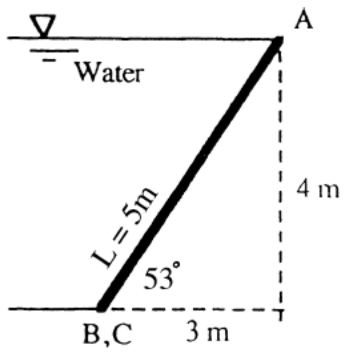
Problems on Forces on surfaces under fluid, Buoyant force, Stability, and Relative equilibrium

1 :

Panel ABC in the slanted side of a water tank (shown at right) is an isoceles triangle with vertex at A and base BC = 2 m. Find the water force on the panel and its line of action.

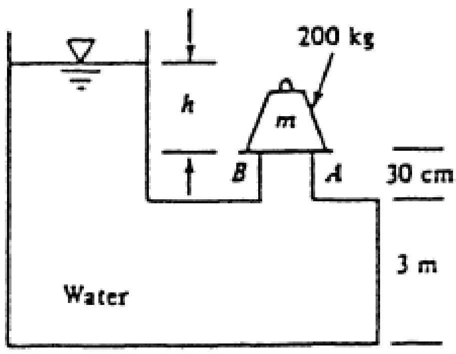


Ans: a- F=131000 N,

b- The center of pressure is 3.75 m down from A, or 1.25 m up from BC.

2:

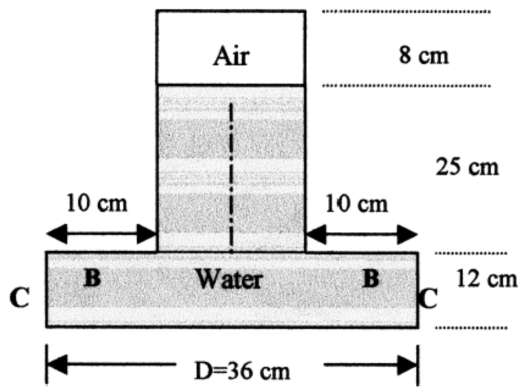
In Fig. weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level *h* will dislodge the gate?



Ans: h = **0.40 m**

3.

The pressure in the air gap is 8000 Pa gage. The tank is cylindrical. Calculate the net hydrostatic force (a) on the bottom of the tank; (b) on the cylindrical sidewall CC; and (c) on the annular plane panel BB.



Ans:

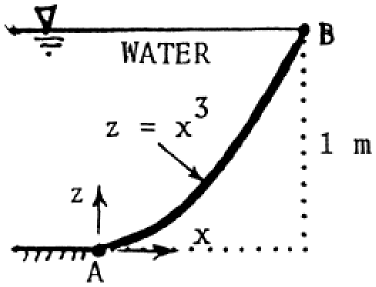
(a) 1180 N.

(b) The net force on the cylindrical sidewall CC= zero due to symmetry.

(c) 853 N

4:

Determine (a) the total hydrostatic force on curved surface AB in Fig. and (b) its line of action. Neglect atmospheric pressure and assume unit width into the paper.



Ans: (a) **8825 N** acting at **56.31°** down and to the right

(b) The line of action of F strikes the vertical above point A at 0.933 m above A, or 0.067 m below the water surface.

Solved problem:1

2.113 A *spar buoy* is a rod weighted to float vertically, as in Fig. P2.113. Let the buoy be maple wood (SG = 0.6), 2 in by 2 in by 10 ft, floating in seawater (SG = 1.025). How many pounds of steel (SG = 7.85) should be added at the bottom so that *h* = 18 in?

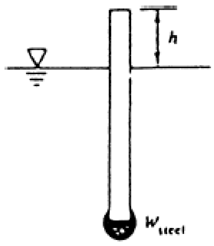


Fig. P2.113

Solution: The relevant volumes needed are

$$\text{Spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(10) = 0.278\ \text{ft}^3; \quad \text{Steel volume} = \frac{W_{steel}}{7.85(62.4)}$$

$$\text{Immersed spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(8.5) = 0.236\ \text{ft}^3$$

The vertical force balance is: buoyancy $B = W_{wood} + W_{steel}$,

$$\text{or:} \quad 1.025(62.4)\left[0.236 + \frac{W_{steel}}{7.85(62.4)}\right] = 0.6(62.4)(0.278) + W_{steel}$$

$$\text{or:} \quad 15.09 + 0.1306 W_{steel} = 10.40 + W_{steel}, \quad \text{solve for} \quad W_{steel} \approx \mathbf{5.4\ lbf} \quad \textit{Ans.}$$

2:

2.112 The uniform 5-m-long wooden rod in the figure is tied to the bottom by a string. Determine (a) the string tension; and (b) the specific gravity of the wood. Is it also possible to determine the inclination angle θ?

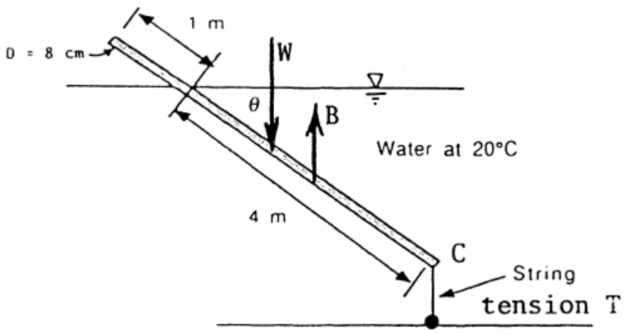


Fig. P2.112

Solution: The rod weight acts at the middle, 2.5 m from point C, while the buoyancy is 2 m from C. Summing moments about C gives

$$\sum M_C = 0 = W(2.5\sin\theta) - B(2.0\sin\theta), \quad \text{or} \quad W = 0.8B$$

$$\text{But} \quad B = (9790)(\pi/4)(0.08\text{ m})^2(4\text{ m}) = 196.8\text{ N.}$$

$$\text{Thus} \quad W = 0.8B = 157.5\text{ N} = SG(9790)(\pi/4)(0.08)^2(5\text{ m}), \quad \text{or:} \quad SG \approx \mathbf{0.64} \quad \textit{Ans.}\text{ (b)}$$

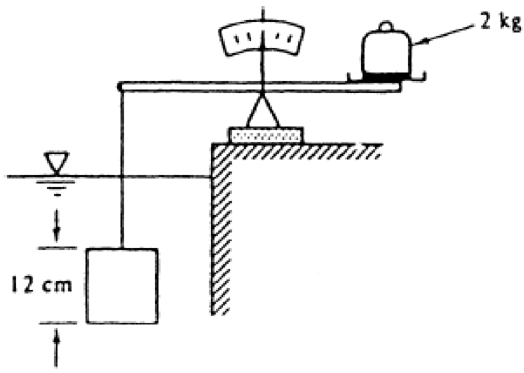
Summation of vertical forces yields

$$\text{String tension} \quad T = B - W = 196.8 - 157.5 \approx \mathbf{39\ N} \quad \textit{Ans.}\text{ (a)}$$

These results are <u>independent of the angle θ</u>, which cancels out of the moment balance.

Pr 5:

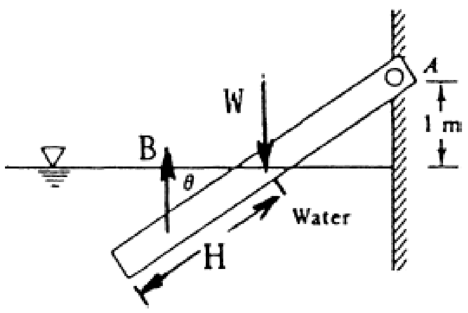
When the 12-cm cube in the figure is immersed in 20°C ethanol, it is balanced on the beam scale by a 2-kg mass. What is the specific gravity of the cube?



Ans: **19100 N/m³**

6:

A uniform wooden beam (SG = 0.65) is 10 cm by 10 cm by 3 m and hinged at A. At what angle will the beam float in 20°C water?



γ_{water}= 9790 N/m³

Ans: θ= 34.3°

Ex 2:

2.119 With a 5-lbf-weight placed at one end, the uniform wooden beam in the figure floats at an angle θ with its upper right corner at the surface. Determine (a) θ; (b) γ_{wood}.

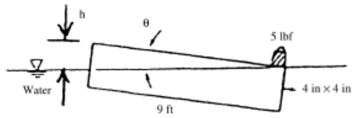


Fig. P2.119

Solution: The total wood volume is $(4/12)^2(9) = 1\ ft^3$. The exposed distance $h = 9\tan\theta$. The vertical forces are

$$\sum F_z = 0 = (62.4)(1.0) - (62.4)(h/2)(9)(4/12) - (SG)(62.4)(1.0) - 5\ lbf$$

The moments of these forces about point C at the right corner are:

$$\sum M_C = 0 = \gamma(1)(4.5) - \gamma(1.5h)(6\ ft) - (SG)(\gamma)(1)(4.5\ ft) + (5\ lbf)(0\ ft)$$

where $\gamma = 62.4\ lbf/ft^3$ is the specific weight of water. Clean these two equations up:

$$1.5h = 1 - SG - 5/\gamma \quad \text{(forces)} \qquad 2.0h = 1 - SG \quad \text{(moments)}$$

Solve simultaneously for $SG \approx \mathbf{0.68}$ *Ans.* (b); $h = 0.16$ ft; $\theta \approx \mathbf{1.02°}$ *Ans.* (a)

Ex 3:

2.134 When floating in water (SG = 1), an equilateral triangular body (SG = 0.9) might take *two* positions, as shown at right. Which position is more stable? Assume large body width into the paper.

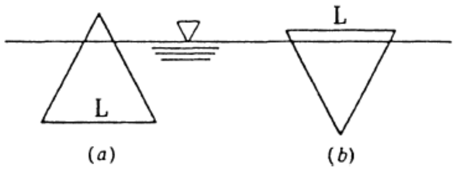


Fig. P2.134

Solution: The calculations are similar to the floating cone of Prob. 2.132. Let the triangle be L by L by L. List the basic results.

(a) Floating with point *up*: Centroid G is 0.289L above the bottom line, center of buoyancy B is 0.245L above the bottom, hence GB = (0.289 – 0.245)L ≈ 0.044L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.0068L = MG + GB = MG + 0.044L$$

$$\text{Hence} \quad MG = \mathbf{-0.037L} \quad \textbf{Unstable} \quad \textit{Ans. (a)}$$

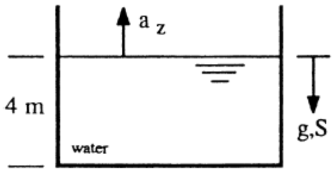
(b) Floating with point *down*: Centroid G is 0.577L above the bottom point, center of buoyancy B is 0.548L above the bottom point, hence GB = (0.577 – 0.548)L ≈ 0.0296L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.1826L = MG + GB = MG + 0.0296L$$

$$\text{Hence} \quad MG = \mathbf{+0.153L} \quad \textbf{Stable} \quad \textit{Ans. (b)}$$

Ex 4:

2.137 A tank of water 4 m deep receives a constant upward acceleration a_z. Determine (a) the gage pressure at the tank bottom if $a_z = 5\ m^2/s$; and (b) the value of a_z which causes the gage pressure at the tank bottom to be 1 atm.



Solution: Equation (2.53) states that $\nabla p = \rho(\mathbf{g} - \mathbf{a}) = \rho(-\mathbf{k}g - \mathbf{k}a_z)$ for this case. Then, for part (a),

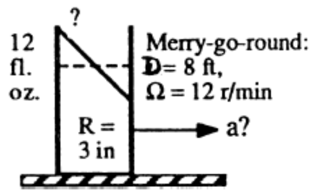
$$\Delta p = \rho(g + a_z)\Delta S = (998\ kg/m^3)(9.81 + 5\ m^2/s)(4\ m) = \mathbf{59100\ Pa\ (gage)} \quad \textit{Ans. (a)}$$

For part (b), we know $\Delta p = 1$ atm but we don't know the acceleration:

$$\Delta p = \rho(g + a_z)\Delta S = (998)(9.81 + a_z)(4.0) = 101350\ Pa \quad \text{if } \mathbf{a_z = 15.6\ \frac{m}{s^2}} \quad \textit{Ans. (b)}$$

Ex 5:

2.138 A 12 fluid ounce glass, 3 inches in diameter, sits on the edge of a merry-go-round 8 ft in diameter, rotating at 12 r/min. How full can the glass be before it spills?



Solution: First, how high is the container? Well, 1 fluid oz. = 1.805 in^3, hence 12 fl. oz. = 21.66 in^3 = $\pi(1.5\ in)^2 h$, or $h \approx 3.06\ in$—It is a fat, nearly square little glass. Second, determine the acceleration toward the center of the merry-go-round, noting that the angular velocity is $\Omega = (12\ rev/min)(1\ min/60\ s)(2\pi\ rad/rev) = 1.26\ rad/s$. Then, for $r = 4$ ft,

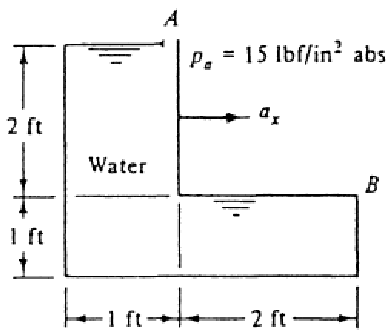
$$a_x = \Omega^2 r = (1.26\ rad/s)^2(4\ ft) = 6.32\ ft/s^2$$

Then, for steady rotation, the water surface in the glass will slope at the angle

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{6.32}{32.2 + 0} = 0.196, \quad \text{or:} \quad \Delta h_{\text{left to center}} = (0.196)(1.5\ in) = 0.294\ in$$

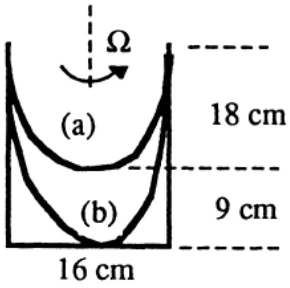
Thus the glass should be filled to no more than 3.06 – 0.294 ≈ 2.77 inches
This amount of liquid is $\upsilon = \pi(1.5\ in)^2(2.77\ in) = 19.6\ in^3 \approx$ **10.8 fluid oz.** *Ans.*

Pr 7: The tank of water in Fig. .143 is full and open to the atmosphere (p_{atm} = 15 psi = 2160 psf) at point A, as shown. For what acceleration a_x, in ft/s^2, will the pressure at point B in the figure be (a) atmospheric; and (b) zero absolute (neglecting cavitation)?



Ex 6:

2.152 A 16-cm-diameter open cylinder 27 cm high is full of water. Find the central rigid-body rotation rate for which (a) one-third of the water will spill out; and (b) the bottom center of the can will be exposed.



Solution: (a) One-third will spill out if the resulting paraboloid surface is 18 cm deep:

$$h = 0.18\text{ m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2(0.08\text{ m})^2}{2(9.81)}, \quad \text{solve for} \quad \Omega^2 = 552,$$

$$\Omega = 23.5\text{ rad/s} = \mathbf{224\ r/min} \quad \textit{Ans.}\text{ (a)}$$

(b) The bottom is barely exposed if the paraboloid surface is 27 cm deep:

$$h = 0.27\text{ m} = \frac{\Omega^2(0.08\text{ m})^2}{2(9.81)}, \quad \text{solve for} \ \ \Omega = 28.8\text{ rad/s} = \mathbf{275\ r/min} \quad \textit{Ans.}\text{ (b)}$$

Ex 7:

2.155 For what uniform rotation rate in r/min about axis C will the U-tube fluid in Fig. P2.155 take the position shown? The fluid is mercury at 20°C.

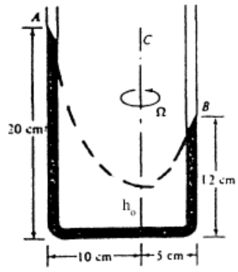


Fig. P2.155

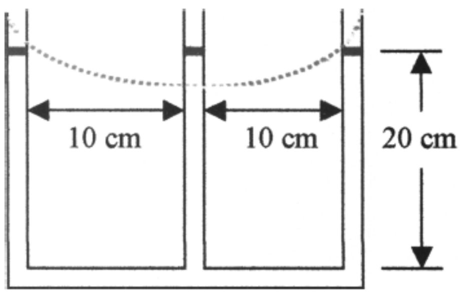
Solution: Let h_o be the height of the free surface at the centerline. Then, from Eq. (2.64),

$$z_B = h_o + \frac{\Omega^2 R_B^2}{2g}; \quad z_A = h_o + \frac{\Omega^2 R_A^2}{2g}; \quad R_B = 0.05\text{ m} \quad \text{and} \quad R_A = 0.1\text{ m}$$

$$\text{Subtract:} \quad z_A - z_B = 0.08\text{ m} = \frac{\Omega^2}{2(9.81)}[(0.1)^2 - (0.05)^2],$$

$$\text{solve} \quad \Omega = 14.5\ \frac{\text{rad}}{\text{s}} = \mathbf{138\ \frac{r}{min}} \quad \textit{Ans.}$$

Pr 8: The three-legged manometer in Fig.) is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity Ω about the central tube, (a) derive a formula to find the change of height in the tubes; (b) find the height in cm in each tube if Ω = 120 rev/min. [*HINT*: The central tube must supply water to *both* the outer legs.]

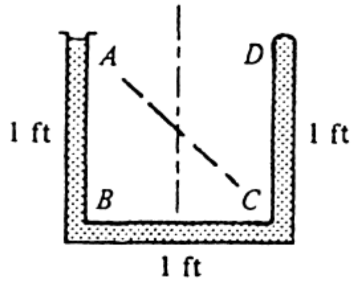


Ans: (a) $-\dfrac{\Omega^2 R^2}{3g}$, (b) $-0.054\ \mathbf{m}\ (down)$

Pr 9:

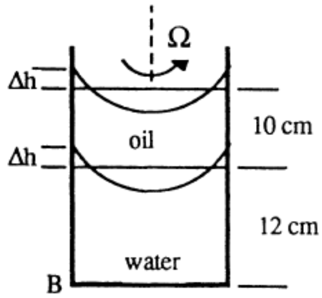
_.-- - The U-tube in Fig. is open at A and closed at D. What uniform acceleration a_x will cause the pressure at point C to be atmospheric? The fluid is water.

Ans: 32.2 ft/s^2



Pr 10:

A very deep 18-cm-diameter can has 12 cm of water, overlaid with 10 cm of SAE 30 oil. It is rotated about the center in rigid-body motion at 150 r/min. (a) What will be the shapes of the interfaces? (b) What and where will be the maximum fluid pressure?



Ans: (a)

same-shape paraboloid, with a deflection Δh up at the wall and down in the center = 5.1 cm

(b) 2550 Pa.