Equally likely outcomes

If the experiment is of such a nature that we can assume equal weights for the sample points of S, i.e "Have same probability to occur"

Ex:- the roll of a fair die, in which the sample space is {1,2,3,4,5,6} and each of these outcomes has probability 1/6

Then the probability of any event A is the ratio of the number of elements in A to the number of elements in S.

P(A)=n(A)/n(S)

Example:

Find the probability of not getting a 3 or 5 while throwing a die.

Sol:

s={1,….,6} and event B={1,2,4,6}then p(B)=n(B)/n(s)=4/6=0.6667.

Note: A and B are complementary events. i.e B=$\bar{A}$

so p(B)=P($\bar{A}$)$= 1 - p(\bar{A})$

For example,

A={3,5}, B={1,2,4,6}

p(B)=P($\bar{A}$)$= 1 - p(\bar{A}) = 1 - 0.333 = 0.6667$

Example

The experience of pulling a number from a box contains the number 1,2,3,4,5,6 and note any number will appear.

A: appear number greater than 3 ={4,5,6}=n(A)=3

B: appear number a smaller than 6={1,2,3,4,5}=n(B)=5

C: appear of an even number {2,4,6}=n(C)=3

Descripts the $\bar{A}$={1,2,3} , $\bar{B}$, $\bar{C}$, $\bar{A}\cap\bar{B}$, $\overline{B \cup A}$,

$\bar{A}$={1,2,3} appear number less than 4,

$\bar{B} = \{6\}$ appear number greater than 5,

$\bar{C}$={ 1,3,5 }

$\bar{A}\cap\bar{B} = \{\,\}$ appear number less than 4 and greater than 5,

$\overline{B \cup A}$ ={ } not greater than 3 or smaller than 6

Example 1

Three items are selected at random from a manufacturing process. Each item is inspected and classified defective (D) or non-defective (N). The sample space providing the most information would be:
S = {NNN, NDN, DNN, NND, DDN, DND, NDD, DDD}
Let B be the event – 'the number of defectives is greater than 1'.

B = {DDN, DND, NDD, DDD},

IF two items S={NN, ND, DN, DD}

C= {only one item is defective} = {ND, DN}

E = {both are non-defective} = {NN}

Example 2
An electronic component is placed on test and we are interested in the time to failure of the component (in hours).

The sample space $S = \{t : 0 \leq t < \infty\}$
Let A be the event 'the component fails before 5 hours' $A = \{t : 0 \leq t < 5\}$.

Example 3
An experiment consists of tipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a dice is tossed.
The sample space S = {(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}
A is the event 'a number less than 4 occurred on the dice',
A =

Example 4
An electronic component is placed on test, let A be the event 'the component fails before 5 hours' and B the event 'the component fails before 10 hours'.

Note the sample space S = $\{t: 0 \leq t < \infty\}$
A = $\{t: 5 \leq t < \infty\}$, B= $\{t: 10 \leq t < \infty\}$.
A∩B = $\{t: 0 \leq t < 5\}$
A∪B = $\{t: 0 \leq t < 10\}$

Example 5

Suppose a dice is tossed, let A be the event ‘an even number turns up’ and B is the event ‘an odd’ number turns up! Then A = {2, 4, 6} B = {1, 3, 5} and $A \cap B = \phi$. Hence A and B are mutually exclusive events.

EXAMPLE 6

When we toss a coin 3 times and record the results in the sequence that they occur, then the sample space is S = { HHH , HHT , HTH , HTT , THH , THT , TTH , TTT } . We may expect each of the 8 outcomes to be equally likely. Thus, the probability of the HTT is 1/8 .

The probability to contain precisely two Heads is 1 /8 + 1 /8 + 1/ 8 = 3/ 8 .

EXAMPLE 7

When we toss a coin 3 times and record the results without paying attention to the order in which they occur, e.g., if we only record the number of Heads, then the sample space is S = { {H, H, H } , {H, H, T } , {H, T, T } , {T, T, T }} . The outcomes in S are now sets; i.e., order is not important. Recall that the ordered outcomes are { HHH , HHT , HTH , HTT , THH , THT , TTH , TTT } .

Note that

{H, H, H } corresponds to one of the ordered outcomes

, {H, H, T } ,, three ,,

{H, T, T } ,, three ,,

{T, T, T } ,, one ,,

Thus {H, H, H } and {T, T, T } each occur with probability 1/ 8 , while {H, H, T } and {H, T, T } each occur with probability 3/ 8.

EXAMPLE 8

If we randomly draw one character from a box containing the characters a, b, and c, then the sample space is S = {a , b , c} , and there are 8 possible events, namely, those in the set of events E = { { } , {a} , {b} , {c} , {a,b} , {a,c} , {b,c} , {a,b,c} }

If the outcomes a, b, and c, are equally likely to occur,

then P({ }) = 0 ,

P({a}) = 1/3 ,

P({b}) = 1/3 ,

P({c}) = 1/3 ,

P({a,b}) = 2/3 ,

P({a,c}) = 2/3 ,

P({b,c}) = 23 ,

P({a,b,c}) = 1 .

EXAMPLE 9

If the sample space is S = {a , b , c , d } ,

and we start with the events E= { { a } , {c, d } } ,

then this set of events needs to be extended to (at least)

E = {{ } , { a } , {c, d } , {b,c, d } , {a,b } , {a,c, d } , { b } , {a,b,c, d } }.

EXERCISE : Verify E includes complements, unions, intersections