Probability Distributions

Lecture 6 **Dr. NOOR H. AYSA**

1- Some Discrete Probability Distributions:
A). Binomial Distributions.
B). Poisson Distributions.
2- Some Continuous Probability Distributions:
Normal Distribution.

Binomial Distributions:

This distribution derived from a process known as a Bernoulli trial.

Bernoulli trial:

Is defined as a single trial of some process or experiment can result in only one of two mutually exclusive outcomes, such as dead or alive, sick or well, male or female. A sequence of Bernoulli trials forms a Bernoulli process under the following conditions:

1- The experiment consists of n repeated trials.
2- Each trial results in one of two possible, mutually exclusive outcomes that may be classified as success or a failure.
3- The probability of success, denoted by P, and probability of failure, denoted by q= (1-p), remains constant from trial to trial.
4- The repeated trials are independent.

Note: The binomial distribution is said to have two parameters, n and *p*.
The probability of successes in n independent trials, is give by

$$P(X) = \binom{n}{x} P^x q^{n-x}$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

Where:

x = 0, 1, 2… n.

n-x = Number of failures

$\binom{n}{x} =$ Coefficient of Binomial

<u>Example:</u>

The probability that a patient recovers from a rare blood disease is 0.4 if 15 people are know to have contracted this disease, what is the probability that; exactly (1), exactly (5), at least (2), from (3 to 5), and between (2 and 5) survive?

$$P(X) = \binom{15}{1} 0.4^{1} \quad 0.6^{15-1}$$

$$= \frac{15!}{1!14!}(0.4)(0.0008) = 0.0048$$

$$p(X) = \binom{15}{5} 0.4^{5} \quad 0.6^{15-5} = 0.1859$$

$$P(X) = 1 - P(X > 2) = 1 - \left[\binom{15}{0} 0.4^{0} \quad 0.6^{15-0} + \binom{15}{1} 0.4^{1} \quad 0.6^{15-1} \right] = 0.9947$$

$$P(3 \le X \le 5) = P(3) + P(4) + P(5)$$

$$= \binom{15}{3} 0.4^{3} \quad 0.6^{15-3} + \binom{15}{4} 0.4^{4} \ 0.6^{15-4} + \binom{15}{5} 0.4^{5} \ 0.6^{15-5} = 0.3760$$

$$P(2 < X < 5) = P(3) + P(4)$$

$$= \binom{15}{3} 0.4^{3} \ 0.6^{15-3} + \binom{15}{4} 0.4^{4} \ 0.6^{15-4} = 0.1267$$

The mean and variance of Binomial distribution are, $\mu = np$ and $\sigma^2 = npq$

From previous example, what are the mean and standard deviation?

$\mu = (15)(0.4) = 6$ And $\sigma^2 = (15)(0.4)(0.6) = 3.6$

$\sigma = \sqrt{\sigma^2} = \sqrt{3.6} = 1.897$

The mean and variance of the proportion of successes of Binomial distribution are,

The mean of the proportion $= \frac{np}{n} = p$

The variance of the proportion $= \frac{npq}{n^2} = \frac{pq}{n}$

Why n^2? Because the variance of *(CX)* $=C^2$ variance *(X),* or the variance of

$(\frac{X}{n}) = \frac{Var(X)}{n^2}$ .... c, and n are constants

Example:

70 out of 200 patients with fractured femur have raised Systolic Blood Pressure (SBP), three months later. What are the mean and the standard deviation for the proportion with raised (SBP) in the population of all fractured femur patients?

$P = \frac{70}{200} = 0.35$ $q = 1 - 0.35 = 0.65$

The mean $= p = 0.35$ $\sigma = \sqrt{\frac{(0.35)(0.65)}{200}} = 0.034$

Poisson distribution:

This distribution is important in describing random occurrences, these occurrences being in either objects in *space or event in time.* The *space* could be a line segment, an area, a volume, or perhaps a piece of material. In this case X might represent the number of bacteria in a given culture, or the number of field mice per acre, the *time* may be any length such as a minute, a day, a week, a month, or even a year. In this case X might represent the number of patients admitted as emergencies to hospital on week days.

The probability distribution of Poisson random variable X is,

$$P(X = x \backslash \lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \text{.................................} \quad x = 0, 1, 2, \ldots$$

$e =$ 2.71828

$\lambda = np$

Where λ (Lambda), is the mean of successes occurring in space or time.
Note: A most important property of Poisson distribution is that is variance equal it means, this distribution applies when is large and p is small, and the occurrence of the event must be independent of the events.

Example:

The probability that a person dies from a certain respiratory infection is 0.002 Find the probability of the next 2000 persons are infected:

1- Exactly 5 persons will die.

2- Fewer than 3 person will die.

$$\lambda = np \qquad \lambda = (2000)(0.002) = 4$$

$$p(X = 5 \backslash 4) = \frac{(2.71828^{-4})(4^{5})}{5!} = 0.1563$$

$$p(X < 3 \backslash 4) = p(X \leq 2 \backslash 4) = p(0) + p(1) + P(2)$$

$$= \frac{(2.71828^{-4})(4^{0})}{0!} + \frac{(2.71828^{-4})(4^{1})}{1!} + \frac{(2.71828^{-4})(4^{2})}{2!} = 0.2381$$