Paths and cycles:

Walk*:* Given a graph G, a *walk* in G is a finite sequence of edges of the form $v_0v_1, v_1v_2, \dots v_{m-1}v_m$, also denoted by $v_0 \to v_1 \to v_2 \to \cdots v_{m-1} \to v_m$, in which any two consecutive edges are adjacent or identical. Such a walk determines a sequence of vertices $v_0, v_1, \dots, v_m$.

We call v_0 the ***initial vertex*** and v_m the ***final vertex*** of the walk, and speak of a walk from v_0 to v_m. The number of edges in a walk is called its ***length***; for example, in *Figur.1.12*,$v \to w \to x \to y \to z \to z \to y \to w$ is a walk of length 7 from v to w.

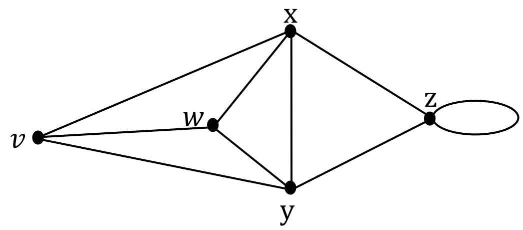

Figure 1.12

The concept of a walk is usually too general for our purposes, so we impose some restrictions.

Trail: A walk in which all the edges are distinct is a *trail*. If, in addition, the vertices $v_0, v_1, \dots, v_m$ are distinct (except, possibly, $v_0 = v_m$) , then the trail is ***a path***.

Note: a walk in which no vertex appears more than once called a path.

Path: A *path* of length $n-1$, denoted by $\boldsymbol{P_n}$, is a sequence of distinct edges $v_1v_2, v_2v_3, \dots, v_{n-1}v_n$.

Note: If $u = v$ then the u, v −walk, u, v −trail and u, v −path is closed.

Cycle: A closed path, a path with $v_1 = v_n$, is called a ***cycle.*** A cycle with n vertices is denoted by $\boldsymbol{C_n}$.

Circuit: A closed trail (without specifying the first vertex) is a *circuit*. A circuit with no repeated vertex is called a cycle.

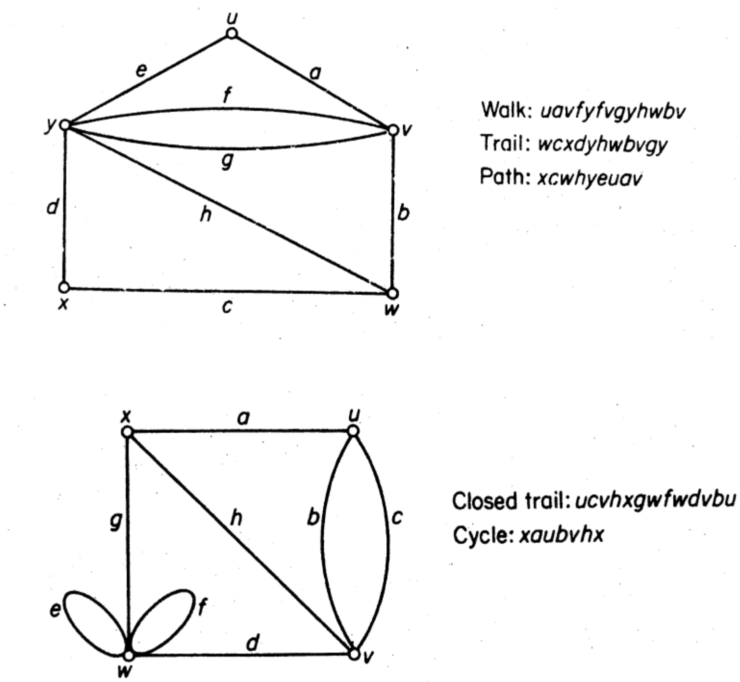

Figure 1.13

1.3 Special Types of Graphs

Well known graphs would be presented here. The importance of these graphs relies in combining or modifying them to create new graphs.

Regular graph: A ***regular graph*** is defined as a graph that all of its vertices are of the same degree, in such case we say that the graph of degree d is a d-regular graph.

Complete graph: A ***complete graph*** of order n (K_n) is a regular graph of degree $n-1$ (as an example, see Figure 1.14).

Null graph: A ***null graph*** is a regular graph of degree zero. Null graph of n vertices is denoted by $\overline{K_n}$**.** (for example, see Figure 1.15).

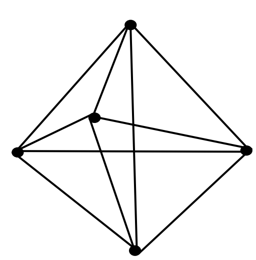

Figure 1.14: K_5

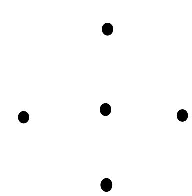

Figure 1.15: $\overline{K_5}$

Bipartite graph: If the vertices set of a graph G can be partitioned into two sets V_1 and V_2 such that any edge of G joins one vertex in V_1 to one vertex in V_2 then G is called a *bipartite graph* having bipartition (V_1, V_2).

Complete bipartite: A ***complete bipartite*** *graph* is a bipartite graph in which each vertex in V_1 is joined to each vertex in V_2. The complete graph having bipartition (V_1, V_2) such that $|V_1| = m$ and $|V_2| = n$ is denoted by $K_{m,n}$.

A bipartite graph of the form $K_{1,n}$ is called a ***star graph*** S_n. Figures 1.16, 1.17 and 1.18 are examples of bipartite graphs

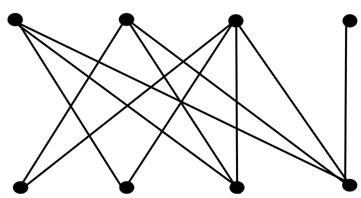

Figure1.16: Bipartite graph

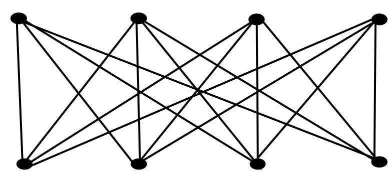

Figure 1.17: $K_{4,4}$

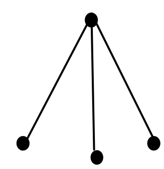

Figure 1.18: S_3

As a similar argument of a bipartite graph, a graph G is *k-partite graph*, $k \geq 3$, if it is possible to partition $V(G)$ into k subsets $V_1, V_2, \dots, V_k$ (called ***partite sets***) such that every element of $E(G)$ joins a vertex of V_i to a vertex of V_j, $i \neq j$.

complete k-partite: A *complete k-partite* graph G is a k-partite graph with partite sets $V_1, V_2, \dots, V_k$, having the added property that if $u \in V_i$ and $v \in V_j$,$i \neq j$, then $uv \in E(G)$. If $|V_i| = n_i$, then this graph is denoted by $K_{n_1, n_2, \dots, n_k}$.

(for instance, see Figure 1.19).

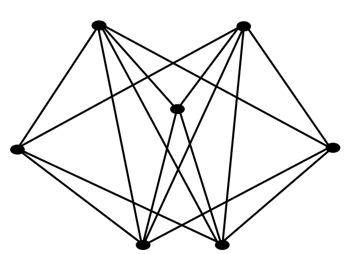

Figure 1.19: $K_{2,3,2}$

Dragon: The *dragon* $D_{n,m}$ is the graph obtained by joining the end point of a path P_m to one vertex of C_n. (for example, see Figure 1.20).

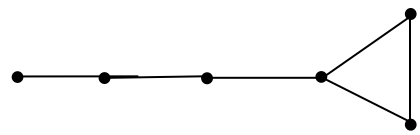

Figure 1.20: $D_{3,4}$

Triangular snake: The *triangular snake* T_n is the graph obtained from the path P_n having the vertices $v_1, v_2, \dots, v_n$ by adding new vertices $w_1, w_2, \dots, w_{n-1}$ and connecting w_i to the vertices v_i, v_{i+1} for each i. (as an example, see Figure 1.21).

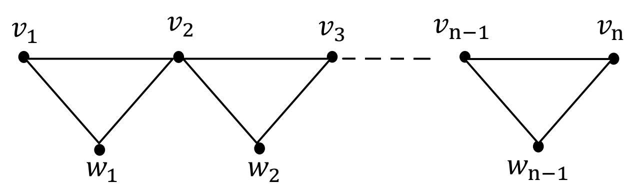

Figure 1.21: Triangular snake T_n