MANOMETERS

Standard Manometers

Manometers are devices that employ liquid columns for determining differences in pressure. The most elementary manometer, usually called a *piezometer*, is illustrated in Fig. 2.10*a*; it measures the pressure in a liquid when it is above zero gage. A glass tube is mounted vertically so that it is connected to the space within the container. Liquid rises in the tube until equilibrium is reached. The pressure is then given by the vertical distance h from the meniscus (liquid surface) to the point where the pressure is to be measured, expressed in units of length of the liquid in the container. It is obvious that the piezometer would not work for negative gage pressures, because air would flow into the container through the tube. It is also impractical for measuring large pressures at A, since the vertical tube would need to be very long. If the specific gravity of the liquid is S, the pressure at A is hS units of length of water.

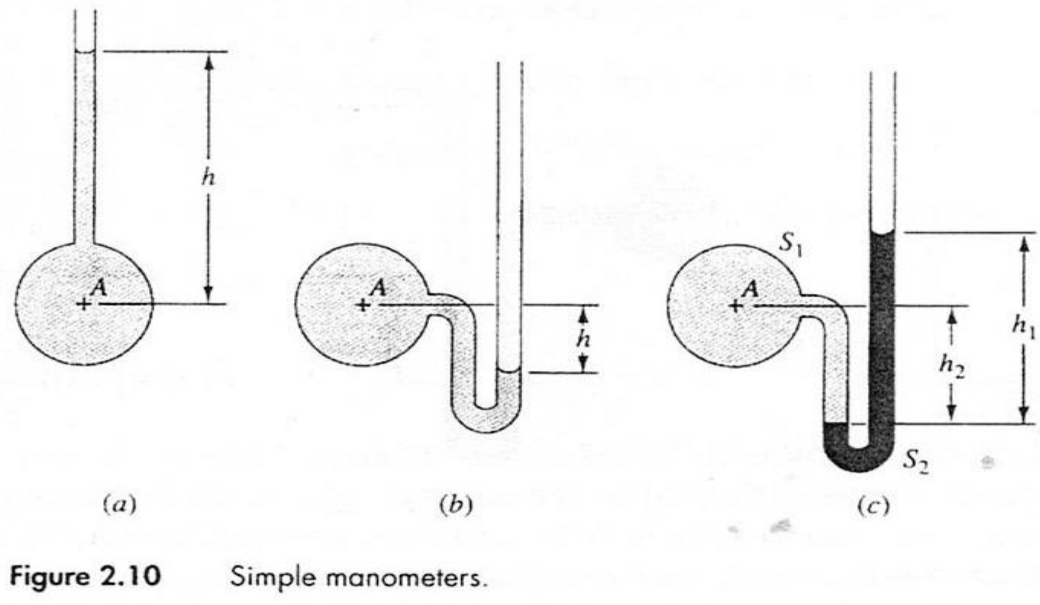


Figure 2.10 Simple manometers.

For measurement of small negative or positive gage pressures in a liquid the tube may take the form shown in Fig. 2.10*b*. With this arrangement the meniscus can come to rest below *A*, as shown. Since the pressure at the meniscus is zero gage and since pressure *decreases* with elevation,

$$h_A = -hS \qquad \text{unit of length of } H_2O$$

For greater negative or positive gage pressures a second liquid of greater specific gravity is employed (Fig. 2.10*c*). It must be immiscible in the first fluid, which may now be a gas. If the specific gravity of the fluid at *A* is S_1 (based on water) and the specific gravity of the manometer liquid is S_2, the equation for pressure at *A* can be written, starting at either *A* or the upper meniscus and proceeding through the manometer, as

$$h_A + h_2S_1 - h_1S_2 = 0$$

in which h_A is the unknown pressure, expressed in length units of water, and h_1 and h_2 are in length units. If *A* contains a gas, S_1 is generally so small that h_2S_1 can be neglected.

A general procedure should be followed in working all manometer problems:

1. Start at one end (or any meniscus if the circuit is continuous) and write the pressure there in an appropriate unit (say pascals) or in an appropriate symbol if it is unknown.
2. Add to this the change in pressure, in the same unit, from one meniscus to the next (plus if the next meniscus is lower and minus if higher).
3. Continue until the other end of the gage (or the starting meniscus) is reached and equate the expression to the pressure at that point, known or unknown.

The expression will contain one unknown for a simple manometer or will give a difference in pressures for the differential manometer. In equation form,

$$p_0 - (y_1 - y_0)\gamma_0 - (y_2 - y_1)\gamma_1 - (y_3 - y_2)\gamma_2 \\ - (y_4 - y_3)\gamma_3 - \cdots - (y_n - y_{n-1})\gamma_{n-1} = p_n$$

in which $y_0, y_1, \cdots, y_n$ are elevations of each meniscus in length units and $\gamma_0, \gamma_1, \gamma_2, \cdots, \gamma_{n-1}$ are specific weights of the fluid columns. The above expression yields the answer in force per unit area and can be converted to other units by use of Fig. 2.8.

hjjj

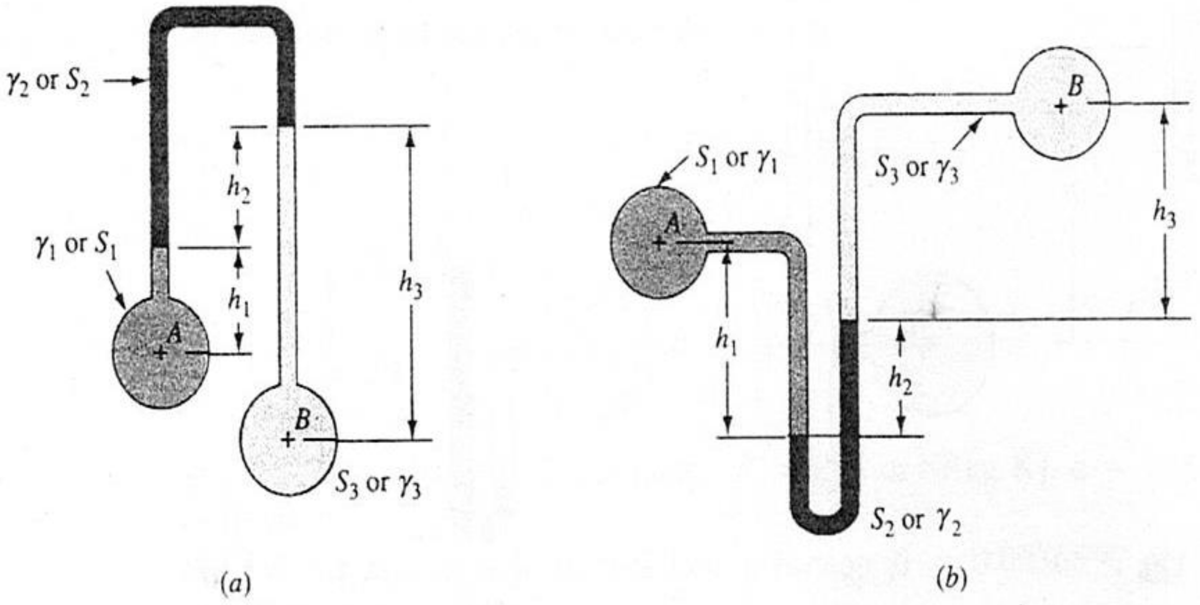


Figure 2.11 Differential manometers.

A differential manometer (Fig. 2.11) determines the difference in pressures at two points *A* and *B* when the actual pressure at any point in the system cannot be determined. Application of the procedure outlined above to Fig. 2.11*a* produces

$$p_A - h_1\gamma_1 - h_2\gamma_2 + h_3\gamma_3 = p_B \quad \text{or} \quad p_A - p_B = h_1\gamma_1 + h_2\gamma_2 - h_3\gamma_3$$

Similarly, for Fig. 2.11*b*,

$$p_A + h_1\gamma_1 - h_2\gamma_2 - h_3\gamma_3 = p_B \quad \text{or} \quad p_A - p_B = -h_1\gamma_1 + h_2\gamma_2 + h_3\gamma_3$$

No formulas for particular manometers should be memorized. It is much more satisfactory to work them out from the general procedure for each case as needed.

If the pressures at *A* and *B* are expressed in length of the water column, the above results can be written, for Fig. 2.11*a*, as

$$h_A - h_B = h_1S_1 + h_2S_2 - h_3S_3 \qquad \text{units of length } H_2O$$

Similarly, for Fig. 2.11*b*

$$h_A - h_B = -h_1S_1 + h_2S_2 + h_3S_3$$

in which S_1, S_2, and S_3 are the applicable specific gravities of the liquids in the system.

Ex.2.5:

In Fig. 2.11*a* the liquids at *A* and *B* are water and the manometer liquid is oil. $S = 0.80$; $h_1 = 300$ mm; $h_2 = 200$ mm; and $h_3 = 600$ mm. (*a*) Determine $p_A - p_B$, in pascals. (*b*) If $p_B = 50$ kPa and the barometer reading is 730-mm Hg, find the pressure at *A* in meters of water absolute.

Solution

(*a*)

$$h_A(\text{m H}_2\text{O}) - h_1 S_{\text{H}_2\text{O}} - h_2 S_{\text{oil}} + h_3 S_{\text{H}_2\text{O}} = h_B(\text{m H}_2\text{O})$$

$$h_A - 0.3(1) - 0.2(0.8) + 0.6(1) = h_B$$

$$h_A - h_B = -0.14\text{-m H}_2\text{O}$$

$$p_A - p_B = \gamma(h_A - h_B) = (9806 \text{ N/m}^3)(-0.14 \text{ m}) = -1373 \text{ Pa}$$

(*b*)

$$h_B = p_B/\gamma = 5 \times 10^4 \text{ N/m}^2/9806 \text{ N/m}^3 = 5.099\text{-m H}_2\text{O}$$

$$h_B(\text{m H}_2\text{O abs}) = h_B(\text{m H}_2\text{O gage}) + (0.73 \text{ m})(13.6)$$

$$= 5.099 + 9.928 = 15.027\text{-m-H}_2\text{O abs}$$

From (a) $$h_{A_{abs}} = h_{B_{abs}} - 0.14 = 15.027 - 0.14 = 14.89\text{-m-H}_2\text{O abs}$$

Micromanometers

Several types of manometers are on the market for determining very small differences in pressure or determining large pressure differences precisely. One type measures the differences in elevation of two menisci of a manometer very accurately. By means of small telescopes with horizontal cross hairs mounted along the tubes on a rack which is raised and lowered by a pinion and slow-motion screw so that the cross hairs can be set accurately, the difference in elevation of menisci (the gage difference) can be read with verniers.

With two immiscible gage liquids a large gage difference *R* (Fig. 2.12) can be produced in the fluid to be measured for a small pressure difference. The heavier gage liquid fills the lower U tube up to 0-0; then the lighter gage liquid is added to both sides, filling the larger reservoirs up to 1-1. The gas or liquid in the system fills the space above 1-1. When the pressure at *C* is slightly greater than at *D*, the menisci move as indicated in Fig. 2.12. The volume of liquid displaced in each reservoir equals the displacement in the U tube; thus,

$$\Delta y A = \frac{R}{2} a$$

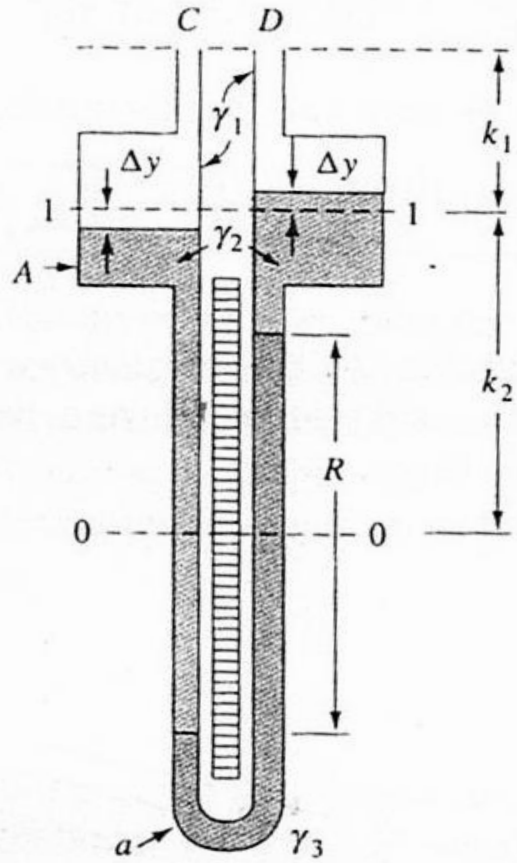


Figure 2.12 Micromanometer using two gage liquids.

in whicl. ... 1 U tube, respectively. The manometer equation can be written, starting at C, in force per unit area, as

$$p_C + (k_1 + \Delta y)\gamma_1 + \left(k_2 - \Delta y + \frac{R}{2}\right)\gamma_2 - R\gamma_3 - \left(k_2 - \frac{R}{2} + \Delta y\right)\gamma_2 - (k_1 - \Delta y)\gamma_1 = p_D$$

in which γ_1, γ_2, and γ_3 are the specific weights as indicated in Fig. 2.12. Simplifying and substituting for Δy gives

$$p_C - p_D = R\left[\gamma_3 - \gamma_2\left(1 - \frac{a}{A}\right) - \gamma_1\frac{a}{A}\right] \tag{2.4.1}$$

The quantity in brackets is a constant for specified gage and fluids; hence, the pressure difference is directly proportional to R.

Ex.2.6

In the micromanometer of Fig. 2.12 find the pressure difference, in pascals, when air is in the system; $S_2 = 1.0$, $S_3 = 1.10$, $a/A = 0.01$, $R = 5$ mm, $T = 20°C$, and the barometer reads 760-mm Hg.

Solution

$$\rho_{air} = \frac{p}{RT} = \frac{(0.76\text{ m})[13.6(9806\text{ N/m}^3)]}{(287\text{ N}\cdot\text{m/kg}\cdot\text{K})(273 + 20\text{ K})} = 1.205\text{ kg/m}^3$$

$$\gamma_1 \frac{a}{A} = (1.205\text{ kg/m}^3)(9.806\text{ m/s}^2)(0.01) = 0.118\text{ N/m}^3$$

$$\gamma_3 - \gamma_2\left(1 - \frac{a}{A}\right) = (9806\text{ N/m}^3)(1.10 - 0.99) = 1079\text{ N/m}^3$$

The term $\gamma_1(a/A)$ can be neglected. Substituting into Eq. (2.4.1) gives

$$p_C - p_D = (0.005\text{ m})(1079\text{ N/m}^3) = 5.39\text{ Pa}$$

The inclined manometer (Fig. 2.13) is frequently used for measuring small differences in gas pressures. It is adjusted to read zero, by moving the inclined scale,

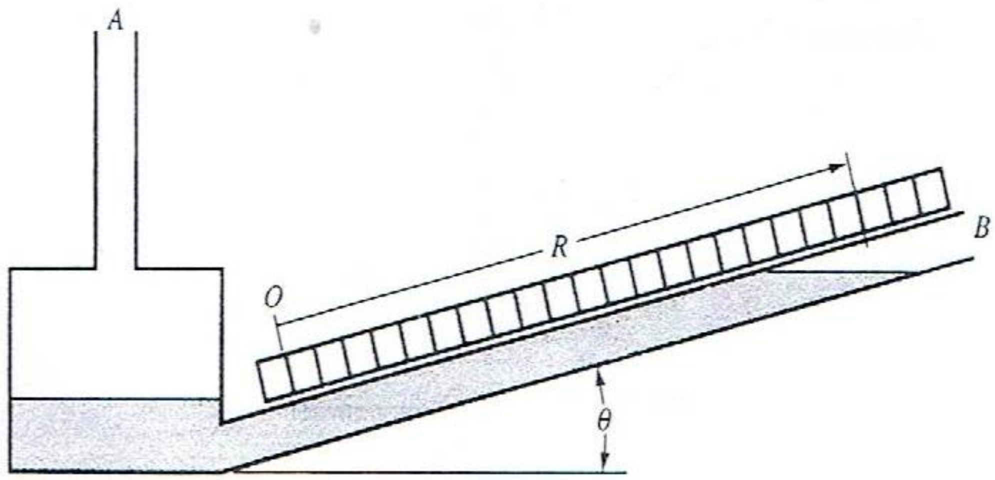


Figure 2.13 Inclined manometer.

when A and B are open. Since the inclined tube requires a greater displacement of the meniscus for a given pressure difference than a vertical tube, it affords greater accuracy in reading the scale.

Surface tension causes a capillary rise in small tubes. If a U tube is used with meniscus in each leg, the surface-tension effects cancel. The capillary rise is negligible in tubes with a diameter of 15 mm or greater.

Application to Manometry:

From the hydrostatic formula (2.20), a change in elevation $z_2 - z_1$ of a liquid is equivalent to a change in pressure $(p_2 - p_1)/\gamma$. Thus a static column of one or more liquids or gases can be used to measure pressure differences between two points. Such a device is called a *manometer*. If multiple fluids are used, we must change the density in the formula as we move from one fluid to another. Figure 2.8 illustrates the use of the formula with a column of multiple fluids. The pressure change through each fluid is calculated separately. If we wish to know the total change $p_5 - p_1$, we add the successive changes $p_2 - p_1$, $p_3 - p_2$, $p_4 - p_3$, and $p_5 - p_4$. The intermediate values of p cancel, and we have, for the example of Fig. 2.8,

$$p_5 - p_1 = -\gamma_0(z_2 - z_1) - \gamma_w(z_3 - z_2) - \gamma_G(z_4 - z_3) - \gamma_M(z_5 - z_4) \quad (2.31)$$

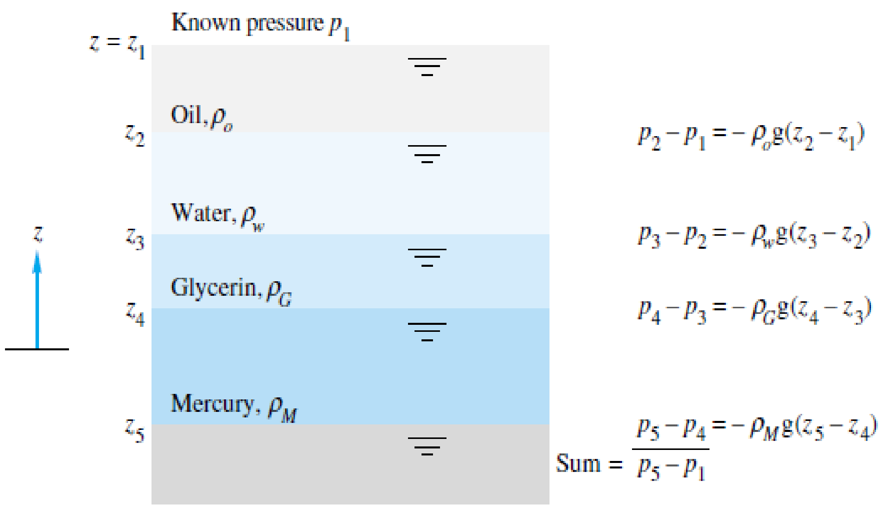


Fig. 2.8 Evaluating pressure changes through a column of multiple fluids.

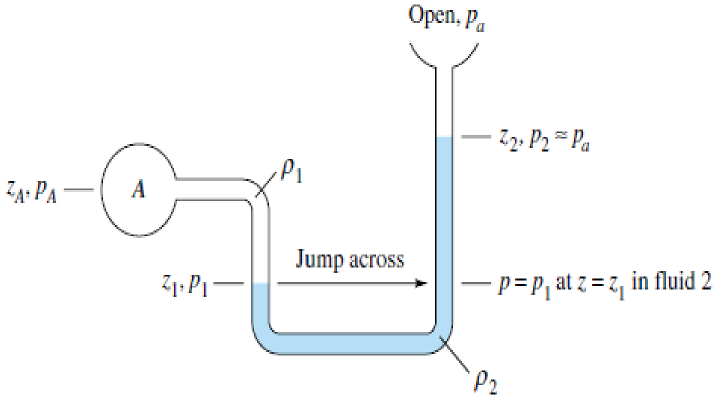


Fig. 2.9 Simple open manometer for measuring p_A relative to atmospheric pressure.

Foss of Michigan State University:

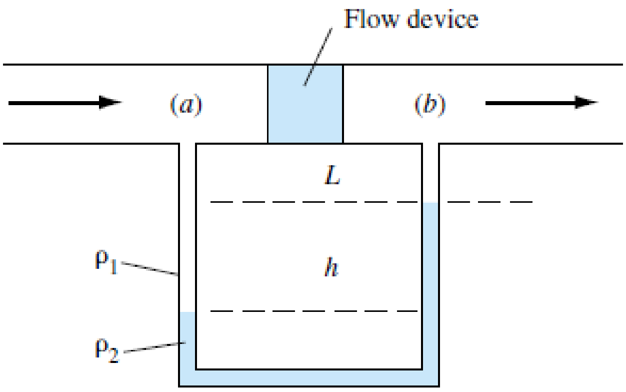
$$p_{down} = p^{up} + \gamma|\Delta z| \tag{2.32}$$

down or up. For example, Eq. (2.31) could be rewritten in the following "multiple increase" mode:

$$p_5 = p_1 + \gamma_0|z_1 - z_2| + \gamma_w|z_2 - z_3| + \gamma_G|z_3 - z_4| + \gamma_M|z_4 - z_5|$$

EXAMPLE 2.3

The classic use of a manometer is when two U-tube legs are of equal length, as in Fig. E2.3, and the measurement involves a pressure difference across two horizontal points. The typical application is to measure pressure change across a flow device, as shown. Derive a formula for the pressure difference *pa - pb* in terms of the system parameters in Fig.



Solution

Using our "up-down" concept as in Eq. (2.32), start at (*a*), evaluate pressure changes around the U-tube, and end up at (*b*):

$pa + \rho_1 gL + \rho_1 gh - \rho_2 gh - \rho_1 gL = pb$

or $\quad pa - pb = (\rho_2 - \rho_1)gh$ **Ans.**

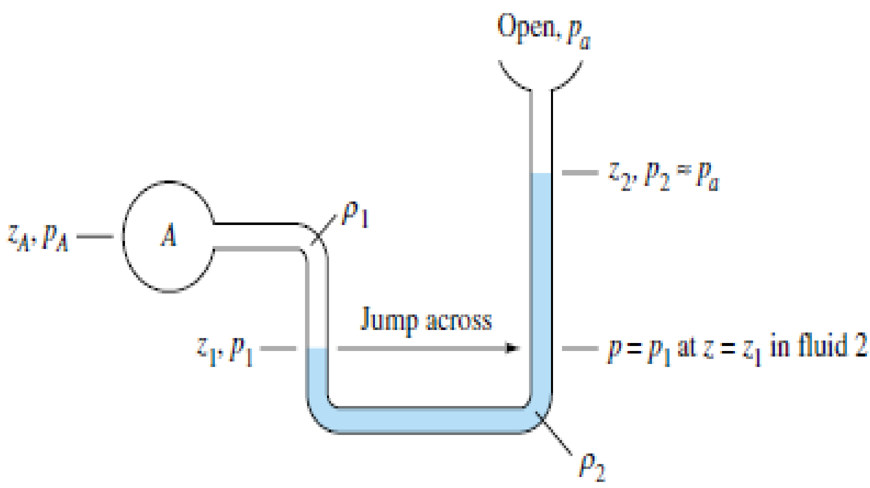
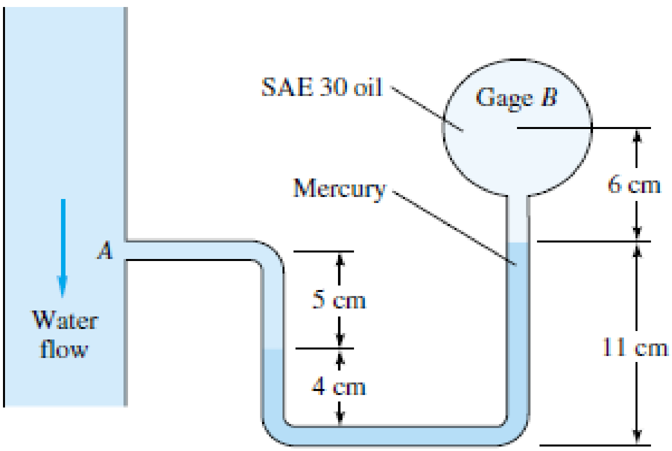


Fig. 2.9 Simple open manometer for measuring p_A relative to atmospheric pressure.

EXAMPLE 2.4

Pressure gage *B* is to measure the pressure at point *A* in a water flow. If the pressure at *B* is 87 kPa, estimate the pressure at *A*, in kPa. Assume all fluids are at 20°C. See Fig. E2.4.



Solution

First list the specific weights from Table 2.1 or Table A.3:

$$\gamma_{water} = 9790 \text{ N/m}^3 \qquad \gamma_{mercury} = 133{,}100 \text{ N/m}^3 \qquad \gamma_{oil} = 8720 \text{ N/m}^3$$

Now proceed from *A* to *B*, calculating the pressure change in each fluid and adding:

$$p_A - \gamma_W(\Delta z)_W - \gamma_M(\Delta z)_M - \gamma_O(\Delta z)_O = p_B$$

or $\quad p_A - (9790 \text{ N/m}^3)(-0.05 \text{ m}) - (133{,}100 \text{ N/m}^3)(0.07 \text{ m}) - (8720 \text{ N/m}^3)(0.06 \text{ m})$

$$= p_A + 489.5 \text{ Pa} - 9317 \text{ Pa} - 523.2 \text{ Pa} = p_B = 87{,}000 \text{ Pa}$$

where we replace N/m^2 by its short name, Pa. The value $\Delta z_M = 0.07$ m is the net elevation change in the mercury (11 cm − 4 cm). Solving for the pressure at point *A*, we obtain

$$p_A = 96{,}351 \text{ Pa} = 96.4 \text{ kPa}$$ *Ans.*

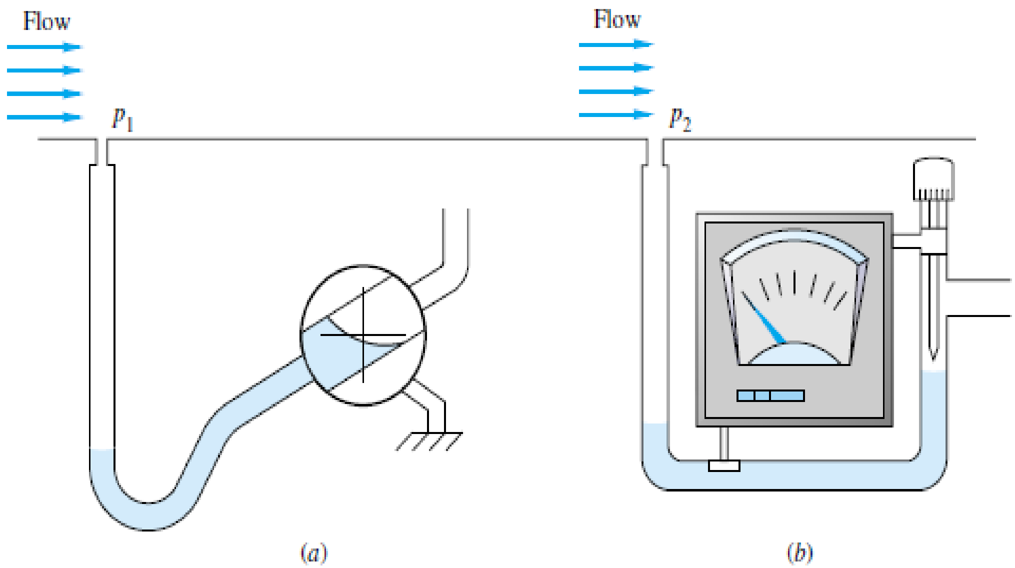


Fig. 2.25 Two types of accurate manometers for precise measurements: (*a*) tilted tube with eyepiece; (*b*) micrometer pointer with ammeter detector.

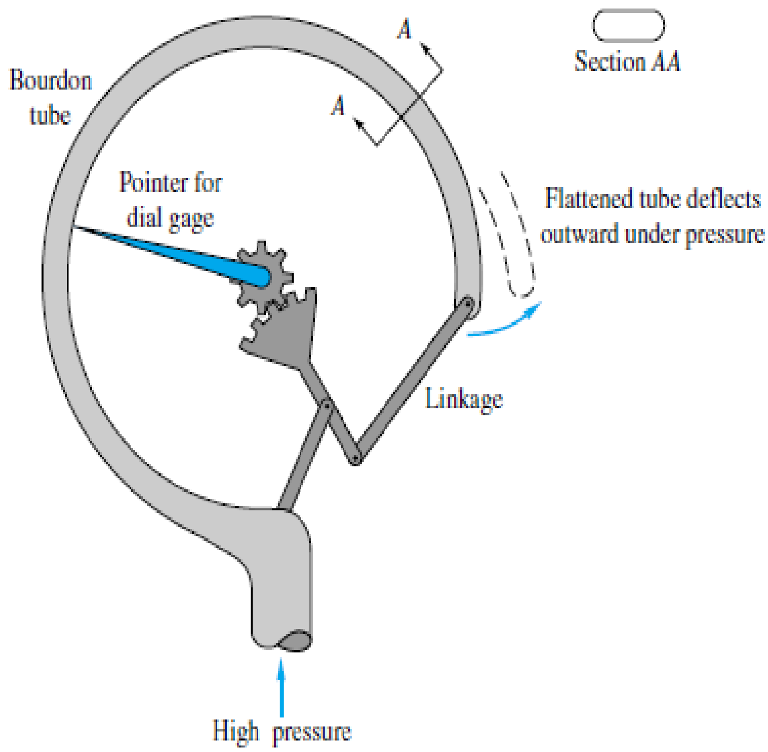
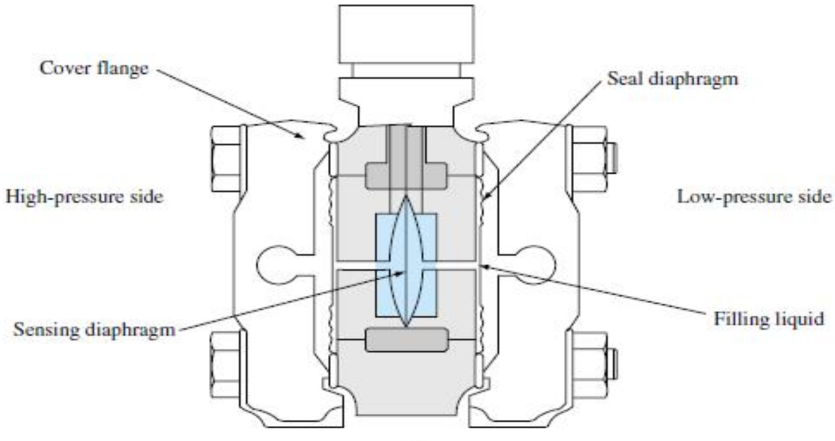
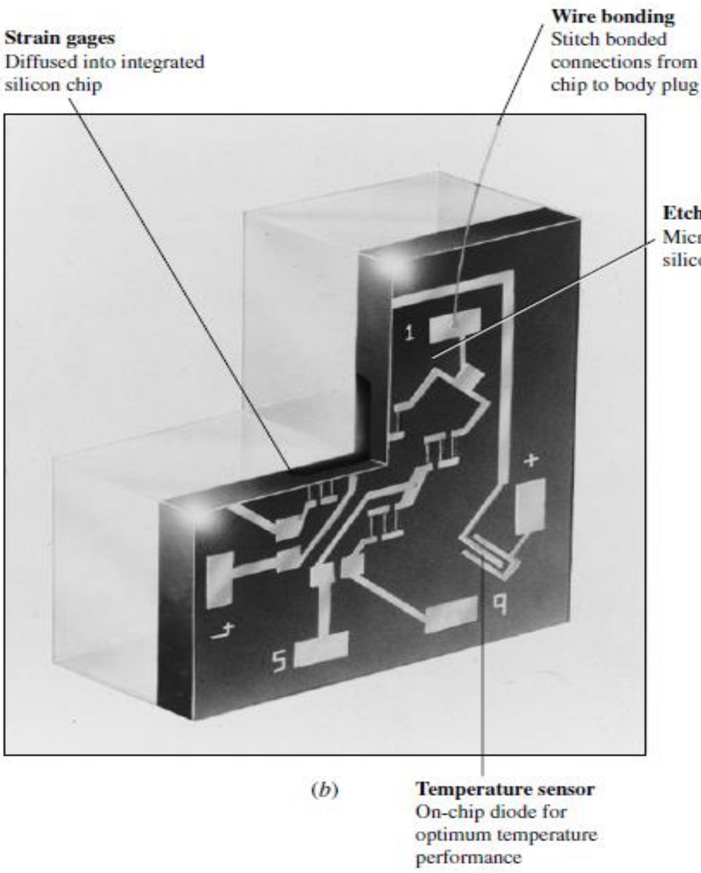


Fig. 2.26 Schematic of a bourdon-tube device for mechanical measurement of high pressures.

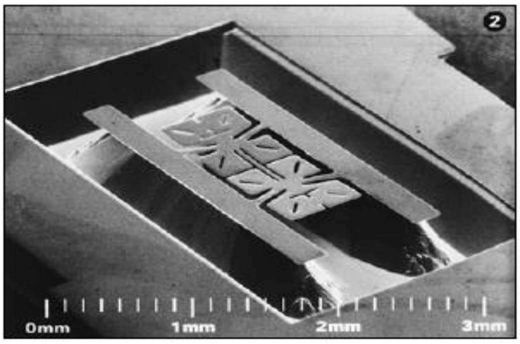


(*a*)



(*b*)

Fig. 2.28 Pressure sensors with electric output: (*a*) a silicon diaphragm whose deflection changes the cavity capacitance (*Courtesy of Johnson-Yokogawa Inc.*); (*b*) a silicon strain gage which is stressed by applied pressure; (*c*) a micromachined silicon element which resonates at a frequency proportional to applied pressure. *[(b) and (c) are courtesy of Druck, Inc., Fairfield, CT.]*



(*c*)

Fig. 2.6: a modern portable barometer, with digital readout, uses the resonating silicon element .

Problems

1. A vertical clean glass piezometer tube has an inside diameter of 1 mm. When a
2. pressure is applied, water at 20°C rises into the tube to a height of 25 cm. After correcting for surface tension, estimate the applied pressure in Pa. For water, let $\sigma = 0.073$ N/m, contact angle $\theta = 0°$, and $\gamma = 9790$ N/m3.
   Ans: **2160 Pa**
3. Atlanta, Georgia, has an average altitude of 1100 ft. On a U.S. standard day, pressure gage A reads 93 kPa and gage B reads 105 kPa. Express these readings in gage or vacuum pressure, whichever is appropriate.
   Ans: Gage A = vacuum.
   Gage B = gage.

4. The deepest point in the ocean is 11034 m in the Mariana Tranch in the Pacific. this depth γ seawater = 10520 N/m3. Estimate the absolute pressure at this depth.Ans: **1121 atm**

5

2.12 In Fig. P2.12 the tank contains water and immiscible oil at 20°C. What is h in centimeters if the density of the oil is 898 kg/m^3?

8.0 cm *Ans.*

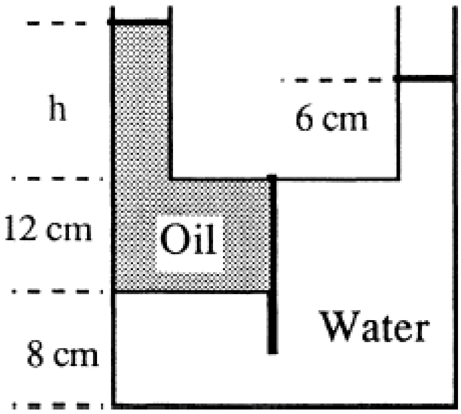


Fig. P2.12