4.2.4 *Holdup*

Because of the considerable differences in the physical properties (particularly viscosity and density) of gases and liquids, the gas always tends to flow at a higher average velocity than the liquid. Sometimes, this can also occur if the liquid preferentially wets the surface of the pipe and therefore experiences a greater drag. In both cases, the volume fraction (holdup) of liquid at any point in a pipe will be greater than that in the mixture entering or leaving the pipe. Furthermore, if the pressure falls significantly along the pipeline, the holdup of .liquid will progressively decrease as a result of the expansion of the gas

If α_L and α_G are the holdups for liquid and gas respectively, it follows that:

$$\alpha_L + \alpha_G = 1 \tag{4.1}$$

Similarly for the input volume fractions:

$$\lambda_L + \lambda_G = 1 \tag{4.2}$$

λ_L and λ_G may be expressed in terms of the flow rates Q_L and Q_G, at a given point in the pipe, as:

$$\lambda_L = \frac{Q_L}{Q_L + Q_G} = \frac{V_L}{V_L + V_G} \tag{4.3}$$

$$\lambda_G = \frac{Q_G}{Q_L + Q_G} = \frac{V_G}{V_L + V_G} \tag{4.4}$$

where V_L and V_G are the superficial velocities of the two phases. Only under the limiting conditions of no-slip between the two phases and of no significant pressure drop along the pipe will α and λ be equal. Liquid (or gas) holdup along the length of the pipe must be known for the calculation of two-phase pressure drop.

Experimental determination

The experimental techniques available for measuring holdup fall into two categories, namely, direct and indirect methods. The direct method of measurement involves suddenly isolating a section of the pipe by means of quick-acting valves and then determining the quantity of liquid trapped. Good reproducibility may be obtained, as shown by widespread use of this technique for both Newtonian and non-Newtonian liquids [Hewitt *et al.*, 1963; Oliver and Young-Hoon, 1968; Mahalingam and Valle, 1972; Chen and Spedding, 1983]. It yields a volume average value of holdup. Although the method is, in principle, simple, it has two main drawbacks. Firstly, the valves cannot operate either instantaneously or exactly simultaneously. This must lead to inaccuracies and, after each measurement, ample time must be allowed for the flow to reach a steady state. Secondly, it is not practicable to use this method for high temperature and pressure situations and/or when either the gas or the liquid or both is of hazardous nature.

The indirect non-intrusive methods have the advantage of not disturbing the flow. The underlying principle is to measure a physical or electrical property that is strongly dependent upon the composition of the gas–liquid mixture. Typical examples include the measurement of γ-ray or X-ray attenuation [Petrick and Swanson, 1958; Pike *et al*; 1965; Shook and Liebe, 1976], or of change in impendence [Gregory and Mattar, 1973; Shu *et al.*, 1982] or of change in conductivity [Fossa, 1998]. Such methods, however, require calibration and yield values (averaged over the cross-section) at a given position in the pipeline. The γ-ray attenuation method has been used extensively to measure liquid holdup for two-phase flow of mixtures of air and non-Newtonian liquids such as polymer solutions and particulate suspensions in horizontal and vertical pipelines [Heywood and Richardson, 1979; Farooqi and Richardson, 1982; Chhabra *et al.*, 1984; Khatib and Richardson, 1984].

Predictive methods for horizontal flow

Methods available for the prediction of the average value of liquid holdup fall into two categories: those methods which are based on models which utilise information implicit in the flow pattern and those which are entirely empirical. Taitel and Dukler [1980] have developed a semi-theoretical expression for the average liquid holdup and the two-phase pressure gradient for the stratified flow of mixtures of air and Newtonian liquids. Although such analyses attempt to give some physical insight into the flow mechanism, they inevitably entail gross simplifications and empiricism. For instance, Taitel and Dukler [1980] assumed the interface to be smooth and the interfacial friction factor to be the same as that for the gas, but this model tends to underestimate the two-phase pressure drop. This methodology has subsequently been extended to the stratified flow of a gas and power-law liquids [Heywood and Charles, 1979]. Similar idealised models are available for the annular flow of gas and power-law liquids in horizontal pipes [Eissenberg and Weinberger, 1979] but most of them assume the liquid to be in streamline flow and the gas turbulent.

The second category of methods includes purely empirical correlations which disregard the flow patterns and are applicable over stated ranges of the variables. Although such an approach contributes little to our understanding, it does provide the designer with the vital information of a known degree of accuracy and reliability. Numerous empirical expressions are available in the literature for the prediction of the liquid holdup when the liquid is Newtonian and these have been critically evaluated [Mandhane *et al.*, 1975; Govier and Aziz, 1982; Chen and Spedding, 1986]. The simplest and perhaps most widely used correlation is that of Lockhart and Martinelli [1949] which utilises the pressure drop values for single phase flow to define a so-called Lockhart–Martinelli parameter, χ which is:

$$\chi = \left(\frac{-\Delta p_L/L}{-\Delta p_G/L}\right)^{1/2} \tag{4.5}$$

where $(-\Delta p_L/L)$ and $(-\Delta p_G/L)$ are, respectively, the pressure gradients for the flow of liquid and gas alone at the same volumetric flowrates as in the two-phase flow. Although it is based on experimental data for the flow of air–water mixtures in small diameter tubes (~25 mm) at near atmospheric pressure and temperature, this correlation has proved to be quite successful when applied to other fluids and for tubes of larger diameters. The original correlation, shown in Figure 4.4, consistently over-estimates the value of the liquid holdup (α_L) in horizontal flow of two-phase gas-Newtonian liquid mixtures. This can be

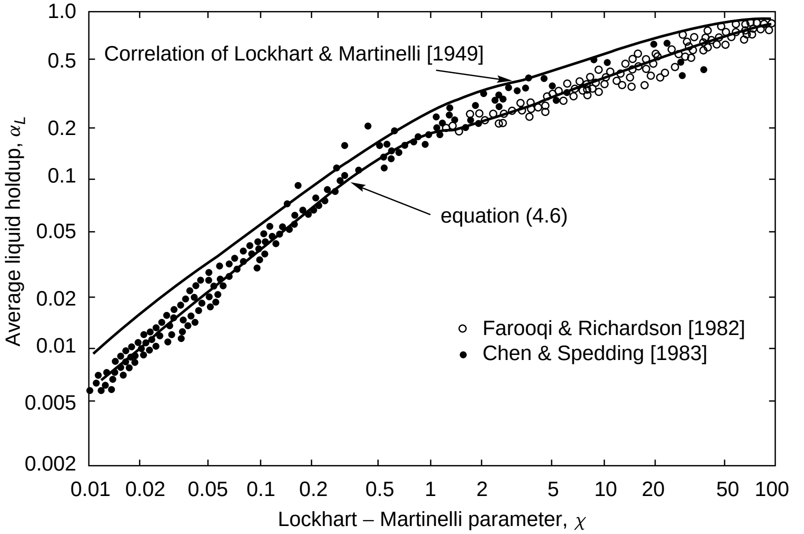

Figure 4.4 *Lockhart–Martinelli correlation for liquid holdup and representative experimental results*

seen in Figure 4.4 which shows the comprehensive data [Chen and Spedding, 1983] for air-water mixtures, of Farooqi [1981] and of Farooqi and Richardson [1982] for the flow of air with aqueous glycerol solutions of various compositions. Taken together, the experimental results shown in Figure 4.4 cover a range of four orders of magnitude of the Lockhart–Martinelli parameter, χ, and average liquid holdups from 0.5% to $\sim$100%. These data cover all the major flow patterns and flow regimes, e.g. nominal streamline and turbulent flow of both gas and liquid. The available experimental results are well represented by the following empirical expressions, as shown in Figure 4.4:

$$\alpha_L = 0.24\chi^{0.8} \qquad 0.01 \le \chi \le 0.5 \tag{4.6a}$$

$$\alpha_L = 0.175\chi^{0.32} \qquad 0.5 \le \chi \le 5 \tag{4.6b}$$

$$\alpha_L = 0.143\chi^{0.42} \qquad 5 \le \chi \le 50 \tag{4.6c}$$

$$\alpha_L = \frac{1}{0.97 + \dfrac{19}{\chi}} \qquad 50 \le \chi \le 500 \tag{4.6d}$$

Furthermore, these equations predict values of α_L to within $\pm$1% at the values of χ marking the changeover point between equations. The overall average error is of the order of 7% and the maximum error is about 15%.

Gas–non-Newtonian systems

Because of the widely different types of behaviour exhibited by non-Newtonian fluids, it is convenient to deal with each flow regime separately, depending upon whether the liquid flowing on its own at the same flow rate would be in streamline or turbulent flow. While it is readily conceded that streamline flow does not have as straightforward a meaning in two phase flows as in the flow of single fluids, for the purposes of correlating experimental results, the same criterion is used to delineate the type of flow for non-Newtonian fluids as discussed in Chapter 3 (Section 3.3), and it will be assumed here that the flow will be streamline for $\mathrm{Re}_{MR} < 2000$, prior to the introduction of gas.

Streamline flow of liquid

The predictions from equation (4.6) will be compared first with the experimental values of average liquid holdup for cocurrent two-phase flow of a gas and shear-thinning liquids. For a liquid of given rheology (m and n), the pressure gradient $(-\Delta p_L/L)$ may be calculated using the methods presented in Chapter 3 but only the power-law model will be used here.

Figure 4.5 shows representative experimental results for average values of liquid holdup α_L, as a function of the parameter χ, together with the predictions of equation (4.6). The curves refer to a series of aqueous china clay suspensions

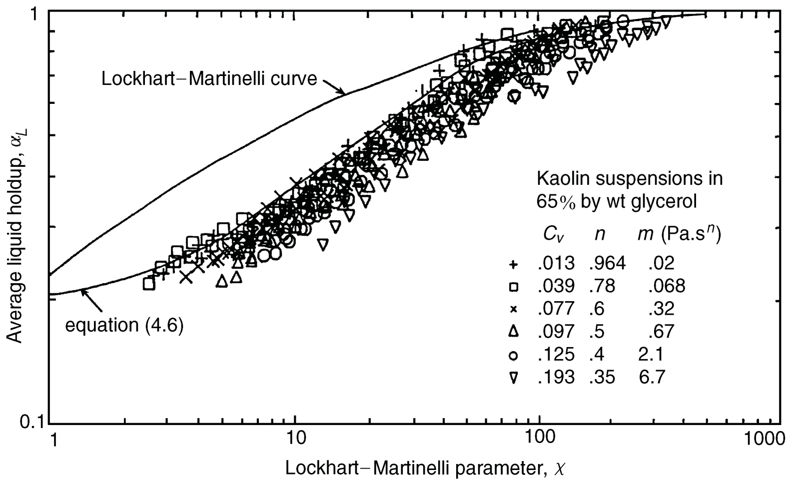

Figure 4.5 *Average liquid holdup data for kaolin suspensions in 65% aqueous glycerol solution in streamline flow (D = 42 mm)*

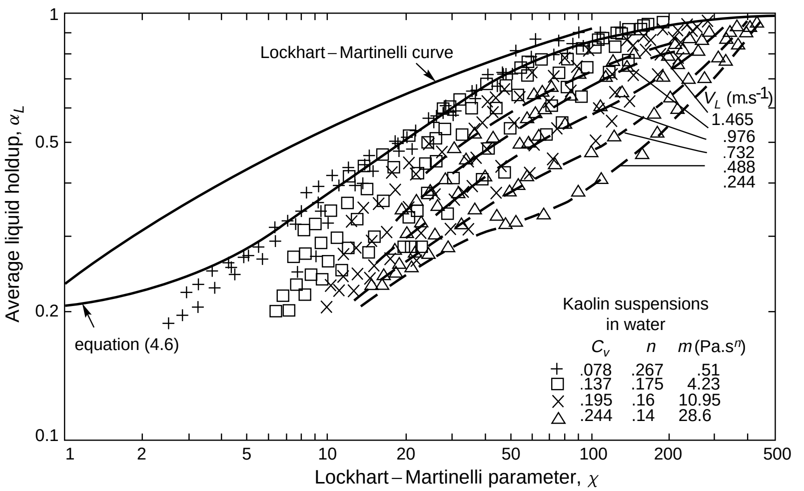

Figure 4.6 *Effect of superficial liquid velocity an average liquid holdup*

in co-current flow with air in a 42 mm diameter horizontal pipe. In addition, Figure 4.6 clearly shows the influence of the liquid superficial velocity (V_L) on the average values of liquid holdup. The results in Figures 4.5 and 4.6 show a similar functional relationship between α_L and χ to that predicted by equation (4.6), but it is seen that this equation over-estimates the value of the average liquid holdup, and distinct curves are obtained for each value of the power-law index (n) and the superficial velocity of the liquid. A detailed examination of the voluminous experimental results reported by different investigators [Oliver and Young-Hoon, 1968; Farooqi and Richardson, 1982; Chhabra and Richardson, 1984] reveals the following features:

(i) For a given value of the power-law index (n), the lower the value of the liquid superficial velocity (V_L), the lower is the average liquid holdup (see Figure 4.6).
(ii) The average liquid holdup decreases as the liquid becomes more shear-thinning (i.e. lower value of n), and the deviation from the Newtonian curve becomes progressively greater.

This suggests that any correction factor which will cause the holdup data for shear-thinning fluids to collapse onto the Newtonian curve, must become progressively smaller as the liquid velocity increases and the flow behaviour index, n, decreases. Based on such intuitive and heuristic considerations, Farooqi and Richardson [1982] proposed a correction factor, J, to be applied to the Lockhart–Martinelli parameter, χ, so that a modified parameter χ_{mod} is defined as:

$$\chi_{\text{mod}} = J\chi \tag{4.7}$$

where
$$J = \left(\frac{V_L}{V_{L_c}}\right)^{1-n} \tag{4.8}$$

and the average liquid holdup is now given simply by replacing χ with χ_{mod} in equation (4.6), viz.:

$$\alpha_L = 0.24(\chi_{\text{mod}})^{0.8} \qquad 0.01 \le \chi_{\text{mod}} \le 0.5 \tag{4.9a}$$

$$\alpha_L = 0.175(\chi_{\text{mod}})^{0.32} \qquad 0.5 \le \chi_{\text{mod}} \le 5 \tag{4.9b}$$

$$\alpha_L = 0.143(\chi_{\text{mod}})^{0.42} \qquad 5 \le \chi_{\text{mod}} \le 50 \tag{4.9c}$$

$$\alpha_L = \frac{1}{0.97 + \dfrac{19}{\chi_{\text{mod}}}} \qquad 50 \le \chi_{\text{mod}} \le 500 \tag{4.9d}$$

V_{L_c} is the critical velocity for the transition from laminar to turbulent flow. For a given power-law liquid (i.e. known m and n), density and pipe diameter, D, V_{L_c} may be estimated simply by setting the Reynolds number (equation 3.8b) equal to 2000, i.e.

$$\text{Re}_{MR} = \frac{\rho V_{L_c}^{2-n} D^n}{8^{n-1} m \left(\dfrac{3n+1}{4n}\right)^n} = 2000 \tag{4.10}$$

For both, $V_L = V_{L_c}$ and/or $n = 1$, the correction factor $J = 1$.

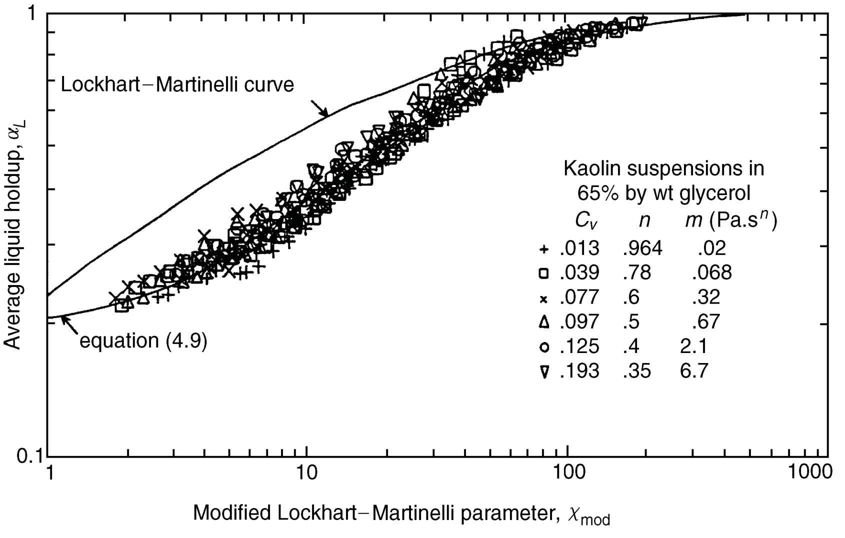

Figure 4.7 *Average liquid holdup as a function of modified parameter* χ_{mod}

In Figure 4.7, the experimentally determined values of average liquid holdup, α_L, are plotted against the modified parameter χ_{mod} for suspensions of kaolin in aqueous glycerol (same data as shown in Figure 4.5) and it will be seen that they are now well correlated by equation (4.9).

Equally good correlations are obtained for the experimental data for two-phase flow of air and nitrogen with aqueous and non-aqueous suspensions of coal [Farooqi *et al.*, 1980] and china clay particles and aqueous solutions of a wide variety of chemically different polymers [Chhabra *et al.*, 1984]. A wide range has been covered ($0.14 \le n \le 1$; $0.1 \le \chi_{mod} \le \sim 200$) but most data have been obtained in relatively small diameter (3 mm to 50 mm) pipes [Chhabra and Richardson, 1986].

Little is known about the influence of visco-elastic properties of the liquid phase on liquid holdup [Chhabra and Richardson, 1986]. However, Chhabra *et al.* [1984] used aqueous solutions of polyacrylamide (Separan AP-30) as model visco-elastic liquids and a preliminary analysis of these results indicated that equation (4.9) consistently underestimated the value of liquid holdup. Infact the experimental results for visco-elastic liquid and air lie between those predicted by equation (4.9) and equation (4.6). It is thus necessary to introduce an additional parameter to account for visco-elastic effects. For this purpose, a Deborah number was defined as:

$$\mathrm{De} = \frac{\lambda_f V_M}{D} \tag{4.11}$$

where λ_f, the fluid characteristic time, is deduced from the measurement of the primary normal stress difference N_1. Like viscosity, it is generally possible to approximate the variation of N_1 with the shear rate over a limited range by a power m_1 law, eg. equation (1.27) i.e.

$$N_1 = m_1(\dot{\gamma})^{p_1} \tag{4.12}$$

which, in turn, allows the fluid characteristic time λ_f to be defined by equation (1.26):

$$\lambda_f = \quad \frac{m_1}{2m}^{\quad 1/(p_1-n)} \tag{4.13}$$

Although the use of the no-slip mixture velocity, V_m, in equation (4.11) is quite arbitrary, it does account for the enhanced shearing of the liquid brought about by the introduction of gas into the pipeline. Over the range of conditions ($0.3 \le \text{De} \le 200$ and $2 \le \chi_{\text{mod}} \le 160$), the following simple expression provides a reasonably satisfactory correlation of the available data for α_{LV}, the average value of liquid holdup for visco-elastic liquids:

$$\alpha_{LV} = \alpha_L \quad 1 + 0.56\frac{\text{De}^{0.05}}{\chi_{\text{mod}}^{0.5}} \tag{4.14}$$

α_L is the value of holdup from equation (4.9) in the absence of visco-elastic effects. Although, equation (4.14) does reduce to the limit of $\alpha_{LV} = \alpha_L$ as $\text{De} \to 0$, extrapolation outside the range quoted above must be carried out with caution.

Transitional and turbulent flow of liquids ($\text{Re}_{MR} > 2000$)

When the non-Newtonian liquid is no longer in streamline flow (prior to the addition of gas), i.e. $\text{Re}_{MR} > 2000$, the experimental results for average liquid holdup agree well with those predicted by equation (4.6) and the original Lockhart–Martinelli parameter χ may therefore be used. This is confirmed by the data shown in Figure 4.8 for a variety of shear-thinning liquids including polymer solutions, chalk–water slurries, china clay and coal suspensions [Raut and Rao, 1975; Farooqi *et al*., 1980; Farooqi and Richardson, 1982; Chhabra *et al*., 1984]. Scant results available in the literature suggest that equation (4.6) underpredicts the value of holdup for visco-elastic liquids in turbulent flow [Rao, 1997].

Predictive methods for upward vertical flow

The previous discussion on holdup related only to horizontal flow of gas – non-Newtonian liquid mixtures. Very few experimental results are available for holdup in vertical upward flow with shear-thinning liquids [Khatib and Richardson, 1984]. These authors used a γ-ray attenuation method

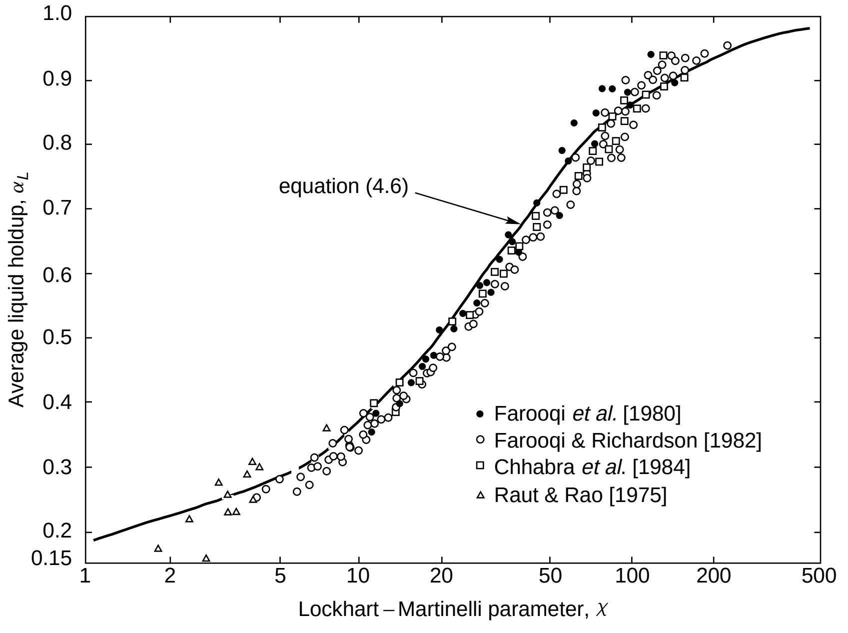

Figure 4.8 *Experimental and predicted (equation 4.6) values of liquid holdup under turbulent conditions for liquid*

to measure the average as well as instantaneous values of liquid holdup for shear-thinning suspensions of china clay and air flowing upwards in a 38 mm diameter pipe. The average values of liquid holdup in streamline flow are in line with the predictions from equation (4.9).

Thus, in summary, average liquid holdup can be estimated using equation (4.6) for Newtonian liquids under all flow conditions, and for non-Newtonian liquids in transitional and turbulent regimes ($Re_{MR} > 2000$).

For the streamline flow of shear-thinning fluids ($Re_{MR} < 2000$), it is necessary to use equation (4.9). A further correction must be introduced (equation 4.14) for visco-elastic liquids. Though most of the correlations are based on horizontal flow, preliminary results indicate that they can also be applied to the vertical upward flow of mixtures of gas and non-Newtonian liquids.