

## **5.6 Flow through packed beds of particles (porous media)**

The flow of non-Newtonian liquids through beds of particles is treated in an analogous way to that adopted in Chapter 3 for the flow through ducts of regular cross-section. No complete analytical solution is, however, possible and a degree of empiricism complemented by the use of experimental results is often necessary. Firstly, however, the basic nature and structure of porous media (or beds of particles) will be briefly discussed.

### **5.6.1 Porous media**

The simplest way of regarding a porous medium is as a solid structure with passages through which fluids can flow. Most naturally occurring minerals (sand, limestones) are consolidated having been subjected to compressive forces for long times. Packed beds of glass beads, catalyst particles, Raschig rings, berl saddles, etc. as used in process equipment are unconsolidated. Unconsolidated media generally have a higher permeability and offer less resistance to flow. Packing may be ordered or random according to whether or not there is a discernable degree of order of the particles, though completely random packing hardly ever occurs as 'order' tends to become apparent as the domain of examination is progressively reduced in size. Cakes and breads are good examples of random media!

Porous media may be characterized at two distinct levels: microscopic and macroscopic. At the microscopic level, the structure is expressed in terms of a statistical description of the pore size distribution, degree of inter-connection and orientation of the pores, fraction of dead pores, etc. In the macroscopic approach, bulk parameters are employed which have been averaged over scales much larger than the size of pores. These two approaches are complementary and are used extensively depending upon the objective. Clearly, the microscopic description is necessary for understanding surface phenomena such as adsorption of macromolecules from polymer solutions and the blockage of pores, etc., whereas the macroscopic approach is often quite adequate for process design where fluid flow, heat and mass transfer are of greatest interest, and the molecular dimensions are much smaller than the pore size. Detailed accounts

of micro- and macro-level characterization methods frequently used for porous media are available in the literature [Greenkorn, 1983; Dullien, 1992].

Of the numerous macroscopic parameters used to quantify porous media, those gaining widest acceptance in the literature for describing the flow of single phase fluids are voidage, specific surface, permeability and tortuosity. Their values can often be inferred from experiments on the streamline flow of single phase Newtonian fluids.

### (i) Voidage

Voidage,  $\varepsilon$ , is defined as the fraction of the total volume which is free space available for the flow of fluids, and thus the fractional volume of the bed occupied by solid material is  $(1 - \varepsilon)$ . Depending upon the nature of the porous medium, the voidage may range from near zero to almost unity. For instance, certain rocks, sandstones etc. have values of the order of 0.15–0.20 whereas fibrous beds and ring packings may have high values of voidage up to 0.95. Obviously, the higher the value of voidage, the lower is the resistance to flow of a fluid.

### (ii) Specific surface

In addition, the specific surface,  $S_B$ , of the bed affects both its general structure and the resistance it offers to flow. It is defined as the surface area per unit volume of the bed, i.e.  $m^2/m^3$ . Hence,  $S_B$  can be expressed in terms of the voidage  $\varepsilon$  and the specific area  $S$  of the particles

$$S_B = S(1 - \varepsilon) \quad (5.28)$$

where  $S$  is the specific area per unit volume of a particle. Thus for a sphere of diameter  $d$ ,

$$S = \frac{\pi d^2}{\pi d^3/6} = \frac{6}{d} \quad (5.29)$$

For a given shape,  $S$  is inversely proportional to the particle size. Highly porous fibre glasses have specific surface areas in the range  $5-7 \times 10^4 \text{ m}^2/\text{m}^3$  while compact limestones ( $\varepsilon \sim 0.04-0.10$ ) have specific surface areas in the range  $\sim 0.2-2 \times 10^6 \text{ m}^2/\text{m}^3$ .

### (iii) Permeability

The permeability of a porous medium may be defined by means of the well-known Darcy's law for streamline flow of an incompressible Newtonian fluid

$$\frac{Q}{A} = V_c = \left( \frac{k}{\mu} \right) \left( -\frac{\Delta p}{L} \right) \quad (5.30)$$

where  $Q$  is the volume rate of flow of a fluid of viscosity,  $\mu$ , through a porous medium of area  $A$  (normal to flow) under the influence of the pressure gradient  $(-\Delta p/L)$ , and  $k$  is called the permeability of the porous medium.

A porous material is said to have a permeability of 1 darcy if a pressure gradient of 1 atm/cm results in a flow of 1 cm<sup>3</sup>/s of a fluid having viscosity of 1 cP through an area of 1 cm<sup>2</sup>. In S.I. units, it is expressed as m<sup>2</sup> and 1 darcy  $\simeq 10^{-12}$  m<sup>2</sup>. Evidently, the lower the permeability, the greater is the resistance to flow. Typical values of permeability range from 10<sup>-11</sup> m<sup>2</sup> for fibre glass to 10<sup>-14</sup> m<sup>2</sup> for silica powder and limestone.

#### (iv) *Tortuosity*

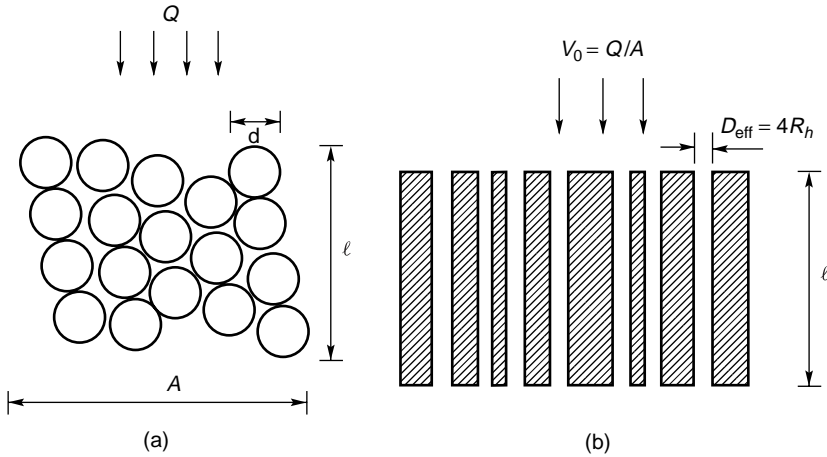
Tortuosity is a measure of the extent to which the path traversed by fluid elements deviates from a straight-line in the direction of overall flow and may be defined as the ratio of the average length of the flow paths to the distance travelled in the direction of flow. Though the tortuosity depends on voidage and approaches unity as the voidage approaches unity, it is also affected by particle size, shape and orientation in relation to the direction of flow. For instance, for plate like particles, the tortuosity is greater when they are oriented normal to the flow than when they are packed parallel to flow. However, the tortuosity factor is not an intrinsic characteristics of a porous medium and must be related to whatever one-dimensional flow model is used to characterise the flow.

### **5.6.2 Prediction of pressure gradient for flow through packed beds**

Many attempts have been made to obtain general relations between pressure drop and mean velocity of flow through porous media or packings, in terms of the bed voidage which is either known or can easily be measured. The following discussion is limited primarily to the so-called capillary tube bundle approach while the other approaches of treating the flow of both Newtonian and non-Newtonian fluids are described in the literature [Happel and Brenner, 1965; Greenkorn, 1983; Dullien, 1992; Chhabra, 1993a].

#### (i) *Streamline flow*

The interstitial void space in a porous matrix or bed of particles may be envisaged as consisting of tortuous conduits of complex cross-section but having a constant average area for flow. Thus, flow in a porous medium is equivalent to that in a non-circular conduit offering the same resistance to flow. However, the flow passages in a bed of particles will be oriented and inter-connected in an irregular fashion and the elementary capillary models do not account for these complexities. Despite these difficulties, the analogy between flow through a circular tube and through the channels in a bed of particles, shown schematically in Figure 5.8, provides a useful basis for deriving a general flow rate – pressure drop expression. As seen in Chapter 3, such expressions vary according to the flow model chosen for the fluid. Following Kemblowski *et al.*



**Figure 5.8** Schematic representation of flow through a bed of uniform spheres (a) and the capillary model idealization (b)

[1987], we begin by re-writing the well-known Hagen–Poiseuille equation, for the streamline flow of an incompressible Newtonian fluid through circular tubes, as follows:

$$V = \frac{D^2}{32\mu} \left( \frac{-\Delta p}{L} \right) \quad (5.31)$$

Then, for flow through a non-circular duct, this may be re-arranged as

$$V = \frac{D_h^2}{16K_0\mu} \left( \frac{-\Delta p}{L} \right) \quad (5.32)$$

where  $D_h$  is the hydraulic mean diameter ( $4 \times$  Pore volume/surface area of particles) and  $K_0$  is a constant which depends only on the shape of the cross-section. For a circular tube of diameter  $D$ , for instance  $D_h = D$  and  $K_0 = 2$ . Likewise, for flow in between two plates separated by a distance  $2b$ ,  $D_h = 4b$  and  $K_0 = 3$ . Kemblowski *et al.* [1987] have further re-arranged equation (5.32) as:

$$\frac{D_h}{4} \left( \frac{-\Delta p}{L} \right) = \mu \left( \frac{4K_0 V}{D_h} \right) \quad (5.33)$$

For an incompressible Newtonian fluid,  $(D_h/4) \frac{-\Delta p}{L}$  is the average shear stress at the wall of the flow passage and  $(4K_0 V/D_h)$ , the shear rate at the

wall, which may be regarded as the nominal shear rate for time-independent non-Newtonian fluids. Thus,

$$\langle \tau_w \rangle = \frac{D_h}{4} \left( \frac{-\Delta P}{L} \right) \quad (5.34)$$

and

$$\langle \dot{\gamma}_w \rangle_n = \frac{4K_0 V}{D_h} \quad (5.35)$$

where  $\langle \rangle$  denotes the values averaged over the perimeter of the conduit.

Equations (5.34) and (5.35) can be used to obtain expressions for streamline flow of time-independent fluids through beds of particles, in which case  $V$  must be replaced by the mean velocity in the pores or interstices, i.e.  $V_i$  and the length  $L$  is replaced by the average length of the tortuous path,  $L_e$ , traversed by the fluid elements.

For a bed whose structure is independent of its depth, then  $L_e$  and  $L$  will be linearly related, i.e.

$$L_e = TL \quad (5.36)$$

where  $T$  is the tortuosity factor.

The interstitial velocity,  $V_i$ , is related to the superficial velocity  $V_0$  by the Dupuit relation which is based on the following considerations.

In a cube of side,  $l$ , the volume of the voids is  $\epsilon l^3$  and the mean cross-sectional area is the free volume divided by the height, i.e.  $\epsilon l^2$ . The volumetric flow rate through the cube is given by  $V_0 l^2$ , so that the average interstitial velocity  $V_i$  is given by

$$V_i = \frac{V_0 l^2}{\epsilon l^2} = \frac{V_0}{\epsilon} \quad (5.37)$$

Although equation (5.37) is a good approximation for random packings, it does not apply to all regular packings. For instance, for a bed of uniform spheres arranged in cubic packing,  $\epsilon = 0.476$ , but the fractional area varies continuously from 0.215 in a plane across the diameters to unity between successive layers. Furthermore, equation (5.37) implicitly assumes that an element of fluid moving at a velocity  $V_i$  covers a distance  $l$  in the same time as a fluid element of superficial velocity  $V_0$  in an empty tube. This implies that the actual interstitial velocity is likely to be somewhat greater than the value given by equation (5.37). Because an element of fluid in a bed actually travels a distance greater than  $L$ , in the Kozeny–Carman capillary model, a correction is made for this effect as:

$$V_i = \frac{V_0 T}{\epsilon} \quad (5.38)$$

Finally, the hydraulic mean diameter  $D_h$  must be expressed in terms of the packing characteristics. Thus, for a bed of uniform spheres of diameter  $d$ , the hydraulic mean diameter  $D_h$  can be estimated as follows:

$$\begin{aligned} D_h &= \frac{4 \times \text{Flow area}}{\text{Wetted perimeter}} = \frac{4 \times \text{volume of flow channels}}{\text{surface area of packing}} \\ &= \frac{4 \times \frac{\text{Volume of flow channels}}{\text{volume of bed}}}{\frac{\text{surface area of packing}}{\text{volume of bed}}} = \frac{4\varepsilon}{S_B} \end{aligned} \quad (5.39)$$

Substitution from equations (5.28) and (5.29) gives:

$$D_h = \left( \frac{2}{3} \frac{d\varepsilon}{(1-\varepsilon)} \right) \quad (5.40)$$

Note that the wetted surface of the column walls has been neglected, which is justified under most conditions of interest.

Although the early versions of capillary models, namely, the Blake and the Blake–Kozeny models, are based on the use of  $V_i = V_0/\varepsilon$  and  $L_e = LT$ , it is now generally accepted [Dullien, 1992] that the Kozeny–Carman model, using  $V_i = V_0T/\varepsilon$  provides a more satisfactory representation of flow in beds of particles. Using equations (5.36), (5.38) and (5.40), the average shear stress and the nominal shear rate at the wall of the flow passage (equations 5.34 and 5.35) may now be expressed as:

$$\langle \tau_w \rangle = \frac{d\varepsilon}{6(1-\varepsilon)T} \left( \frac{-\Delta p}{L} \right) \quad (5.41)$$

and

$$\langle \dot{\gamma}_w \rangle_n = 6K_0T \left( \frac{1-\varepsilon}{\varepsilon^2} \frac{V_0}{d} \right) \quad (5.42)$$

For generalised non-Newtonian fluids, Kemblowski *et al.* [1987] postulated that the shear stress at the wall of a pore or ‘capillary’ is related to the corresponding nominal shear rate at the wall by a power-law type relation:

$$\langle \tau_w \rangle = m' (\langle \dot{\gamma}_w \rangle_n)^{n'} \quad (5.43)$$

where  $m'$  and  $n'$  are the apparent consistency coefficient and flow behaviour index, respectively, inferred from pressure drop/flow rate data obtained in a packed bed. By analogy with the generalised procedure for streamline flow in circular tubes outlined in Chapter 3, equation (3.30a),  $m'$  and  $n'$  can be linked to the actual rheological parameters. For a truly power-law fluid, for instance;

$$n' = n \quad (5.44a)$$

$$m' = m \left( \frac{3n+1}{4n} \right)^n \quad (5.44b)$$

It should be, however, noted that the Rabinowitsch–Mooney factor of  $((3n + 1)/4n)^n$  is strictly applicable only to cylindrical tubes but the limited results available for non-circular ducts suggest that it is nearly independent of the shape of the conduit cross-section [Miller, 1972; Tiu, 1985]. For instance, the values of this factor are within 2–3% of each other for circular tubes and parallel plates over the range  $0.1 \leq n \leq 1$ .

The cross-section of the channels formed in a bed of spheres would be expected to lie between that of a circular tube and of a plane slit and Kemblowski *et al.* [1987] therefore suggested the use of a mean value of 2.5 for  $K_0$ . Considerable confusion also exists in the literature about the value of the tortuosity factor,  $T$ . Thus, Carman [1956] proposed a value of  $\sqrt{2}$  based on the assumption that the capillaries deviate on average by  $45^\circ$  from the mean direction of flow ( $\cos 45 = 1/\sqrt{2}$ ). On the other hand, if a fluid element follows the surface round the diameter of a spherical particle, the tortuosity factor should equal  $\pi/2$ . Indeed, the values ranging from  $\sim 1$  to 1.65 have been used in the literature for Newtonian fluids [Agarwal and O'Neill, 1988]. Because  $T$  is a function of the geometry of the bed, it has the same value whatever the liquid rheology, provided that it is time-independent, although there is some evidence that  $T$  is weakly dependent on flow rate [Dharamadhikari and Kale, 1985]. This is not surprising because macromolecules have a tendency to adsorb on the walls of the pores and, if the flow rate is high enough, the shearing forces may overcome the surface forces; thus polymer molecules become detached thereby making more space available for flow. Thus, flow passages blocked at low flow rates may open up to flow again at high flow rates. Such unusual effects observed with non-Newtonian fluids in porous media are briefly discussed in a later section in this chapter.

A dimensionless friction factor may be defined as:

$$f = \left( \frac{-\Delta p}{L} \frac{d}{\rho V_0^2} \right) \left( \frac{\varepsilon^3}{1 - \varepsilon} \right) \quad (5.45)$$

Noting  $K_0 = 2.5$  and  $T = \sqrt{2}$ , combining equations (5.41) to (5.45):

$$f = \frac{180}{\text{Re}^*} \quad (5.46)$$

$$\text{where } \text{Re}^* = \frac{\rho V_0^{2-n} d^n}{m(1 - \varepsilon)^n} \left( \frac{4n}{3n + 1} \right)^n \left( \frac{15\sqrt{2}}{\varepsilon^2} \right)^{1-n} \quad (5.47)$$

For a Newtonian fluid,  $n = 1$ , both equations (5.46) and (5.47) reduce to the well known Kozeny–Carman equation. Equation (5.47) correlates most of the literature data on the flow of power-law fluids through beds of spherical particles up to about  $\text{Re}^* \sim 1$ , though most work to date has been carried out

in beds having voidages in the range  $0.35 \leq \varepsilon \leq 0.41$  [Kemblowski *et al.*, 1987; Chhabra, 1993a].

### Bingham plastic fluids

The flow of viscoplastic fluids through beds of particles has not been studied as extensively as that of power-law fluids. However, since the expressions for the average shear stress and the nominal shear rate at the wall, equations (5.41) and (5.42), are independent of fluid model, they may be used in conjunction with any time-independent behaviour fluid model, as illustrated here for the streamline flow of Bingham plastic fluids. The mean velocity for a Bingham plastic fluid in a circular tube is given by equation (3.13):

$$V = \frac{D^2}{32\mu_B} \left( \frac{-\Delta p}{L} \left( 1 - \frac{4}{3}\phi + \frac{1}{3}\phi^4 \right) \right) \quad (3.13)$$

where  $\phi = \tau_0^B / \tau_w$ . This equation can be re-arranged in terms of the nominal shear rate and shear stress at the wall of the pore as:

$$(\dot{\gamma}_w)_n = \frac{8V}{D} = \frac{\tau_w}{\mu_B} \left( 1 - \frac{4}{3}\phi + \frac{1}{3}\phi^4 \right) \quad (5.48)$$

As seen in Chapter 3, the quantity  $(8V/D)$  is the nominal shear rate at the wall (also see equation 5.35, for a circular tube,  $D_h = D$ ,  $K_0 = 2$ ). Substituting for the nominal shear rate and wall shear stress from equations (5.41) and (5.42) in equation (5.48), slight re-arrangement gives:

$$f = \frac{180}{\text{Re}_B F(\phi)} \quad (5.49)$$

where

$$\text{Re}_B = \frac{\rho V_0 d}{\mu_B} \quad (5.50)$$

$$F(\phi) = 1 - \frac{4}{3}\phi + \frac{\phi^4}{3} \quad (5.51)$$

$$\text{and} \quad \phi = \frac{\tau_0^B}{\langle \tau_w \rangle} \quad (5.52)$$

It should be noted that  $T = \sqrt{2}$  and  $K_0 = 2.5$  have been used in deriving equation (5.49). Again for the special case of Newtonian fluids,  $\tau_0^B = 0$  or  $\phi = 0$ ,  $F(\phi) = 1$  and equation (5.49) reduces to the Kozeny–Carman equation. The scant experimental data available on pressure drop for the streamline flow for Bingham plastic fluids ( $\text{Re}_B \sim 1$ ) is consistent with equation (5.49).

This section is concluded by noting that similar expressions for the friction factor have been derived for a range of purely inelastic fluid models and these have been critically reviewed elsewhere [Chhabra, 1993a,b].

(ii) *Transitional and turbulent flow*

Because the apparent viscosity of non-Newtonian systems is usually high, flow conditions rarely extend beyond the streamline flow regime. There is no clear cut value of the Reynolds number marking the end of streamline flow. An examination of the available data indicates that the lower the value of the power-law index, the higher the Reynolds number up to which streamline flow occurs. An important factor is that with a range of pore sizes, some can be in laminar flow and others turbulent. Despite these uncertainties, the value of  $Re^* \sim 5-10$  is a good approximation for engineering design calculations.

The capillary bundle approach has also been extended for correlating data on pressure drop in packed beds of spherical particles in the transitional and turbulent regions. Both Mishra *et al.* [1975] and Brea *et al.* [1976] proposed the following empirical method for estimating the ‘effective viscosity’,  $\mu_{\text{eff}}$ :

$$\mu_{\text{eff}} = m' \left\{ \frac{12V_0(1 - \varepsilon)}{d\varepsilon^2} \right\}^{n-1} \quad (5.53)$$

This is then incorporated in the modified Reynolds number,  $Re'$ , to give

$$Re' = \frac{\rho V_0 d}{\mu_{\text{eff}}(1 - \varepsilon)} \quad (5.54)$$

They assumed the ‘viscous’ and ‘inertial’ components of the pressure drop to be additive, and proposed the following relationship between the friction factor and the modified Reynolds number:

$$f = \frac{\alpha}{Re'} + \beta \quad (5.55)$$

Based on their experimental data for the flow of power-law fluids in packed and fluidised beds of spheres ( $0.7 \leq n \leq 1$ ;  $0.01 \leq Re' \leq 1000$ ;  $0.37 \leq \varepsilon \leq 0.95$ ), Mishra *et al.* [1975] obtained  $\alpha = 150$  and  $\beta = 1.75$ . With these values, equation (5.55) coincides with the well-known Ergun equation for Newtonian fluids [Ergun, 1952]. On the other hand, the data of Brea *et al.* [1976] encompass somewhat wider ranges of the power-law index ( $0.4 \leq n' \leq 1$ ) and Reynolds number ( $0.01 \leq Re' \leq 1700$ ) but a limited range of voidage ( $0.36 \leq \varepsilon \leq 0.40$ ) and they proposed  $\alpha = 160$  and  $\beta = 1.75$ . A close scrutiny of equation (5.53) shows that it is tantamount to using  $K_0 = 2$  (corresponding to a circular cross-section) and  $T = (25/12)$ , and thus the lower value of  $K_0$  is compensated for by the higher value of the tortuosity factor,  $T$ . Although the original papers give mean deviations of 15–16% between the predictions

of equation (5.55) and experimental data, careful inspection of the pertinent graphs reveal maximum deviations of up to 100%.

Based on the re-appraisal of the literature data and new data, the following simplified expression provides a somewhat better representation of the data in packed beds, at least for  $\varepsilon \leq 0.41$  and  $Re^* < 100$  [Chhabra, 1993a]:

$$f = \frac{150}{Re^*} + 1.75 \quad (5.56)$$

Thus, it is suggested that for the flow of shear-thinning fluids in packed beds, equation (5.56) should be used for  $Re^* < 100$ , and for  $\varepsilon > 0.41$  and  $Re^* > 100$  equation (5.55) is preferable, with  $\alpha = 150$  and  $\beta = 1.75$ .

### Example 5.6

Estimate the frictional pressure gradient for the flow of a polymer solution ( $m = 3.7 \text{ Pa}\cdot\text{s}^n$ ,  $n = 0.5$ , density =  $1008 \text{ kg/m}^3$ ) at the rate of  $0.001 \text{ m}^3/\text{s}$  through a 50 mm diameter column packed with 1.5 mm leadshots. The average voidage of the packing is 0.39.

### Solution

Superficial velocity of flow,  $V_0 = (0.001/((\pi/4)(50 \times 10^{-3})^2)) = 0.51 \text{ m/s}$

The Reynolds number of flow:

$$\begin{aligned} Re^* &= \frac{\rho V_0^{2-n} d^n}{m(1-\varepsilon)^n} \left( \frac{4n}{3n+1} \right)^n \frac{15\sqrt{2}}{\varepsilon^2}^{1-n} \\ &= \frac{(1008)(0.51)^{2-0.5}(1.5 \times 10^{-3})^{0.5}}{3.7(1-0.39)^{0.5}} \left( \frac{4 \times 0.5}{3 \times 0.5 + 1} \right)^{0.5} \frac{15\sqrt{2}}{0.39^2}^{1-0.5} \\ &= 52 \end{aligned}$$

$\therefore$  the flow is in the transitional regime. Equation (5.55) or (5.56) may be used. For equation (5.56),

$$f = \frac{150}{Re^*} + 1.75 = \frac{150}{52} + 1.75 = 4.63$$

The pressure gradient,  $\frac{-\Delta p}{L}$ , across the bed is calculated using this value of  $f$  in equation (5.45),

$$\begin{aligned} \frac{-\Delta p}{L} &= \frac{f \rho V_0^2}{d} \left( \frac{1-\varepsilon}{\varepsilon^3} \right) = \frac{(4.63)(1008)(0.51)^2(1-0.39)}{(1.5 \times 10^{-3})(0.39)^3} \\ &= 8\,300\,000 \text{ Pa/m} \quad \text{or} \quad 8.3 \text{ MPa/m.} \end{aligned}$$

For the sake of comparison, the value of  $\frac{-\Delta P}{L}$  using equation (5.55) will also be calculated here. For a power law fluid,

$$m' = m \left( \frac{3n + 1}{4n} \right)^n \quad \text{and} \quad n' = n = 0.5$$

$$\therefore m' = 3.7 \left( \frac{3 \times 0.5 + 1}{4 \times 0.5} \right)^{0.5} = 4.14 \text{ Pa}\cdot\text{s}^n$$

$$\therefore \mu_{\text{eff}} = m' \left\{ \frac{12V_0(1 - \varepsilon)}{d\varepsilon^2} \right\}^{n'-1}$$

$$= 4.14 \left\{ \frac{12 \times 0.51 \times (1 - 0.39)}{(1.5 \times 10^{-3})(0.39)^2} \right\}^{0.5-1} = 0.0324 \text{ Pa}\cdot\text{s}$$

The modified Reynolds number,  $\text{Re}'$ , is evaluated using equation (5.54):

$$\text{Re}' = \frac{\rho V_0 d}{\mu_{\text{eff}}(1 - \varepsilon)} = \frac{(1008)(0.51)(1.5 \times 10^{-3})}{0.0324(1 - 0.39)} = 39.1$$

This value also suggests that flow is in the transition regime. The corresponding friction factor is:

$$f = \frac{150}{\text{Re}'} + 1.75 = \frac{150}{39.1} + 1.75 = 5.59$$

Again using equation (5.45),  $\frac{-\Delta P}{L} = 9.6 \text{ MPa/m}$ .

This value is only about 15% higher than that calculated previously.

The above discussion is limited to the flow of inelastic fluids in unconsolidated beds of particles where the pore size is substantially larger than the characteristic dimensions of the polymer molecules. Interaction effects between the walls of the pore and the polymer molecules are then small. Thus, measuring the relationship between pressure drop and flow rate in a packed bed and in a tube would therefore lead to the prediction of the same rheological properties of the fluid. Visco-elastic effects and other phenomena including blockage of pores, polymer adsorption/retention, etc. observed in beds of low permeability or in consolidated systems will be briefly discussed in Section 5.6.7.