5.2.5 ***Effect of container boundaries***

The problem discussed so far relates to the motion of a single spherical particle in an unbounded, or effectively infinite, expanse of fluid. If other particles are present in the neighbourhood of the sphere, its settling velocity will be influenced and the effect will become progressively more marked as the concentration of particles increases. There are three contributory factors. First, as the particles settle, the displaced liquid flows upwards. Secondly, the particle experiences increased buoyancy force owing to the higher density of the suspension. Finally, the flow pattern of the liquid relative to the particles will be altered thereby affecting the velocity gradients. The sedimentation of concentrated suspensions in non-Newtonian fluids is discussed in section 5.2.6 .while the effect of the vessel walls is discussed here

The walls of the vessel containing the liquid exert an extra retarding effect on the terminal falling velocity of the particle. The upward flow of the displaced liquid, not only influences the relative velocity, but also sets up a velocity profile in the confined geometry of the tube. This effect may be quantified by introducing a wall factor, f, which is defined as the ratio of the terminal falling velocity of a sphere in a tube, V_m, to that in an unconfined liquid, V, viz.,

$$f = \frac{V_m}{V} \tag{5.16}$$

The experimental determination of the setling velocity in an infinite medium requires the terminal falling velocity of a sphere to be measured in tubes of different diameters and then extrapolating these results to $(d/D) = 0$, as shown in Figure 5.4 for a series of plastic spheres falling in a 0.5% Methocel solution. When the settling occurs in the creeping flow region (Re < 1), the measured falling velocity shows a linear dependence on the diameter ratio and can readily be extrapolated to $(d/D) = 0$. The available experimental results in Newtonian and power-law liquids indicate that the wall factor, f, is independent of the

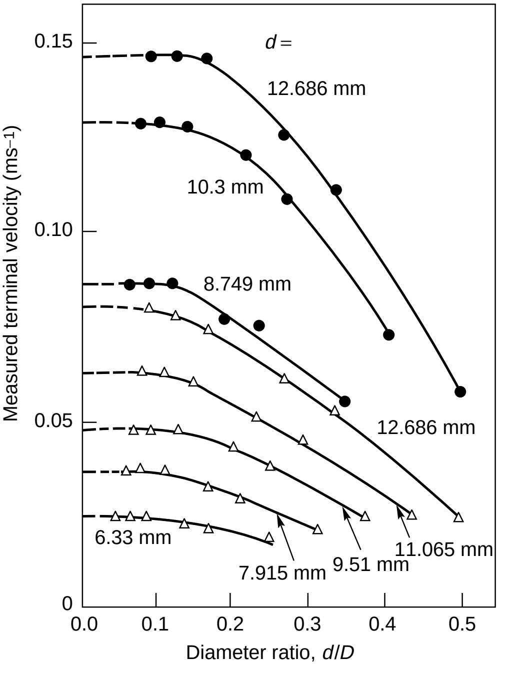

Figure 5.4 *Dependence of terminal falling velocity of spheres in a 0.5% aqueous hydroxyethyl cellulose solution* ($\mathrm{Re}_m > 1$) • *PVC spheres* Δ *Perspex spheres*

sphere Reynolds number (based on the measured velocity, V_m) both at small ($<\sim 1$) and large ($>\sim 1000$) values of the Reynolds number [Chhabra and Uhlherr, 1980; Uhlherr and Chhabra, 1995; Chhabra *et al*., 1996]. Based on an extensive experimental study in the range of conditions $0.5 \le n \le 1$; $0.01 \le \text{Re}_m \le 1000$ and $(d/D) \le 0.5$, the wall factor can be empirically correlated with the diameter ratio and the sphere Reynolds number as [Chhabra and Uhlherr, 1980]:

$$\frac{(1/f) - (1/f_\infty)}{(1/f_0) - (1/f_\infty)} = [1 + 1.3\text{Re}_m^2]^{-1/3} \tag{5.17}$$

where f_0 and f_∞, the values of the wall factor in the low and high Reynolds number regions respectively are given by:

$$f_0 = 1 - 1.6\frac{d}{D} \tag{5.18}$$

and

$$f_\infty = 1 - 3\left(\frac{d}{D}\right)^{3.5} \tag{5.19}$$

While it is readily recognised that the creeping flow occurs up to about $\text{Re}_m \sim 1$, the critical value of the Reynolds number corresponding to the upper asymptotic value, f_∞, is strongly dependent upon the value of (d/D), e.g. ranging from $\text{Re}_m \sim 30$–40 for $(d/D) = 0.1$ to $\text{Re}_m \sim 1000$ for $(d/D) = 0.5$. Qualitatively, the additional retardation caused by the walls of the vessel is less severe is power-law fluids than that in Newtonian fluids under otherwise identical conditions; the effect becoming progressively less important with the increasing Reynolds number and/or decreasing diameter ratio. The wall effect is even smaller in visco-elastic liquids [Chhabra, 1993a].

Sedimenting particles are also subject to an additional retardation as they approach the bottom of the containing vessel because of the influence of the lower boundary on the flow pattern. No results are available on this effect for non-Newtonian fluids and therefore the corresponding expressions for Newtonian fluids offer the best guide [Clift *et al*., 1978], at least for inelastic shear-thinning fluids. For instance, in the creeping flow regime and $d/D < 0.1$, the effect is usually expressed as

$$\frac{V_m}{V} = \frac{1}{1 + 1.65(d/L)} \tag{5.20}$$

where L is the distance from the bottom of the vessel.

5.2.6 Hindered settling

As mentioned earlier, the terminal falling velocity of a sphere is also influenced by the presence of neighbouring particles.

In concentrated suspensions, the settling velocity of a sphere is less than the terminal falling velocity of a single particle. For coarse (non-colloidal) particles in mildly shear-thinning liquids ($1 > n \geq 0.8$) [Chhabra *et al*., 1992], the expression proposed by Richardson and Zaki [1954] for Newtonian fluids applies at values of Re($= \rho V^{2-n} d^n / m$) up to about 2:

$$\frac{V_0}{V} = (1 - C)^Z \tag{5.21}$$

where V_0 is the hindered settling velocity of a suspension of uniform size spheres at a volume fraction C; V is the terminal falling velocity of a single sphere in the same liquid, Z is a constant which is a function of the Archimedes number and (d/D) and is given as [Coulson and Richardson, 1991]:

$$\frac{4.8 - Z}{Z - 2.4} = 0.0365\,\text{Ar}^{0.57}[1 - 2.4(d/D)^{0.27}] \tag{5.22}$$

where for power-law liquids, the Archimedes number, Ar, is defined by equation (5.12).

In visco-elastic fluids, some internal clusters of particles form during hindered settling and the interface tends to be diffuse [Allen and Uhlherr, 1989, Bobbroff and Phillips, 1998]. For 100–200 µm glass spheres in visco-elastic polyacrylamide solutions, significant deviations from equation (5.22) have been observed.

Example 5.5

Estimate the hindered settling velocity of a 25% (by volume) suspension of 200 µm glass beads in an inelastic carboxymethyl cellulose solution ($n = 0.8$ and $m = 2.5\,\text{Pa·s}^n$) in a 25 mm diameter tube. The density of glass beads and of the polymer solution are 2500 kg/m^3 and 1020 kg/m^3 respectively.

Solution

The velocity of a single glass bead is calculated first. In view of the small size and rather high consistency coefficient of the solution, the particle Reynolds number will be low, equation (5.11) can be used. From Table 5.1, $X(n) = 1.24$ corresponding to $n = 0.8$.

Substituting values in equation (5.11):

$$V = \left(\frac{g d^{n+1}(\rho_s - \rho)}{18 m X}\right)^{(1/n)}$$

$$= \left(\frac{9.81 \times (200 \times 10^{-6})^{0.8+1}(2500 - 1020)}{18 \times 2.5 \times 1.24}\right)^{1/(0.8)}$$

$= 4.97 \times 10^{-6}$ m/s or 4.97 µm/s.

Check the value of Reynolds number, Re:

$$\mathrm{Re} = \frac{\rho V^{2-n} d^n}{m} = \frac{1020 \times (4.97 \times 10^{-6})^{2-0.8}(200 \times 10^{-6})^{0.8}}{2.5}$$

$$= 1.93 \times 10^{-7} \ll 1$$

Therefore, the settling occurs in the creeping flow region and the equation (5.11) is valid.

The Archimedes number is given by equation (5.12) as:

$$\mathrm{Ar} = \frac{4}{3} g d^{(2+n)/(2-n)} (\rho_s - \rho) \rho^{n/(2-n)} m^{2/(n-2)}$$

$$= \Bigg(\frac{4}{3} \quad \times 9.81 \times (200 \times 10^{-6})^{(2+0.8)/(2-0.8)}$$

$$\times (2500 - 1020)(1020)^{(0.8)/(2-0.8)}(2.5)^{2/(0.8-2)}$$

$$= 0.000\,996\,4$$

The value of Z is evaluated from equation (5.22):

$$\frac{4.8 - Z}{Z - 2.4} = 0.0365 \times (0.000\,996\,4)^{0.57} \quad 1 - 2.4 \quad \frac{200 \times 10^{-6}}{25 \times 10^{-3}} \quad ^{0.27}$$

$$= 0.000\,247$$

and $\quad Z \simeq 4.8$

$\therefore \quad \dfrac{V_0}{V} = (1 - C)^Z$

or $\quad V_0 = 4.97 \times 10^{-6} \times (1 - 0.25)^{4.8} = 1.25 \times 10^{-6}$ m/s

This velocity is about a quarter of the value for a single particle.

5.3 Effect of particle shape on terminal falling velocity and drag force

A spherical particle is unique in that it presents the same projected area to the oncoming fluid irrespective of its orientation. For non-spherical particles, on the other hand, the orientation must be specified before the drag force can be calculated. The drag force on spheroidal (oblates and prolates) particles moving in shear-thinning and shear-thickening power-law fluids ($0.4 \le n \le 1.8$) have been evaluated for Reynolds numbers up to 100 [6, 7]. The values of drag coefficient are given in the original papers [Tripathi *et al.*, 1994; Tripathi and Chhabra, 1995] and the main trends are summarised here. For pseudoplastic fluids ($n < 1$), creeping flow occurs for Re up to about 1 (based on equal volume sphere diameter) and for dilatant fluids ($n > 1$) up to about 0.2–0.5. For a given Reynolds number and aspect ratio (minor/major axis), the drag on oblates is less than that on a sphere of equal volume whereas for prolate particles, it is higher. The drag force in the creeping flow region is higher for shear-thinning fluids than for Newtonian fluids; this is consistent with the behaviour observed for a sphere. The influence of power-law index, however, diminishes with increasing particle Reynolds number. The opposite effect is observed with shear-thickening fluids, i.e. the drag is lower than that in a Newtonian fluid.

Many workers have measured drag coefficients for particles, including cylinders, rectangular prisms, discs, cones settling at their terminal velocities in power-law fluids. Work in this area has recently been reviewed [Chhabra, 1996], but no generalised correlation has yet been proposed. A simple equation which reconciles the bulk of the results for drag on cones, cubes, parallelpipeds, and cylinders (falling axially) settling at terminal condition in power-law fluids ($\mathrm{Re} < 150$; $0.77 \le n \le 1$; $0.35 \le \psi \le 0.7$) is [Venu Madhav and Chhabra, 1994]:

$$C_D = \frac{32.5}{\mathrm{Re}}(1 + 2.5\mathrm{Re}^{0.2}) \tag{5.23}$$

where both C_D and Re $\left(\dfrac{\rho V^{2-n} d^n}{m}\right.$ are based on the diameter of the sphere of the same volume; corrections were made for wall effects. The predictions deteriorate progressively as the particle departs from spherical shape, i.e. as sphericity, ψ, decreases.

The scant experimental and theoretical results available for viscoplastic and visco-elastic fluids have been reviewed elsewhere [Chhabra, 1996].