3.8.1 *Sudden enlargement*

When the cross-section of a pipe enlarges gradually, the streamlines follow closely the contours of the duct and virtually no extra frictional losses are incurred. On the other hand, whenever the change is sudden, additional losses arise due to the eddies formed as the fluid enters the enlarged cross-section. The resulting head loss for laminar flow can be evaluated by applying the mechanical energy balance in conjunction with the integral momentum balance. Consider the flow configuration shown in Figure 3.23 in which the section ‘2’ is located immediately after the end of the smaller pipe. By suitable choice of plane ‘1’, the frictional pressure loss may be assumed to be negligible between planes ‘1’ and ‘2’ and hence $p_1 \sim p_2$; the latter acts over the whole cross-section (πR_2^2). Also, immediately following the expansion at section ‘2’, the streamlines will be nearly parallel to the axis and the velocity profile at section ‘2’ will be similar to the fully developed profile at section ‘1’. If section ‘3’ is sufficiently far down-stream, the velocity profile again be fully established. On applying the integral momentum balance over the control :volume as shown in Figure 3.23

$$\Sigma F_z = (p_2 - p_3)\pi R_2^2 = -\int_0^{R_1} 2\pi r \rho V_z^2 \, \mathrm{d}r + \int_0^{R_2} 2\pi r \rho V_z^2 \, \mathrm{d}r \qquad (3.92)$$

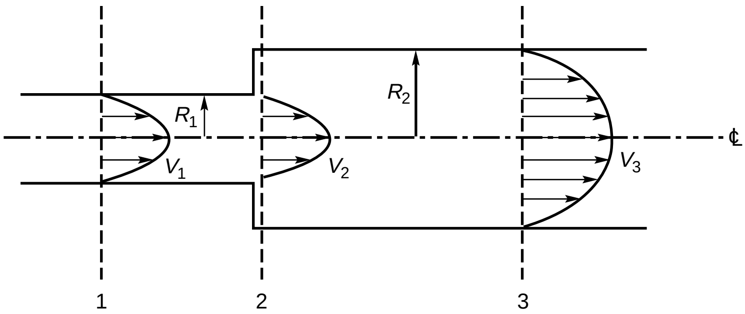

Figure 3.23 *Schematics of laminar flow through a sudden expansion in a tube*

The integration in equation (3.92) can be carried out after insertion of the velocity profiles for the appropriate viscosity model to obtain the pressure loss $(p_2 - p_3)$. For power-law fluids, this procedure leads to:

$$\frac{p_2 - p_3}{\rho} = \left(\frac{3n+1}{2n+1}\right)\left(\frac{Q^2}{A_1^2}\right)\left[\left(\frac{A_1}{A_2}\right)^2 - \frac{A_1}{A_2}\right] \tag{3.93}$$

where $A_1 = \pi R_1^2$ and $A_2 = \pi R_2^2$. Applying the mechanical energy balance equation between points '1' and '3':

$$\frac{p_1}{\rho} + \frac{V_1^2}{2\alpha} + gz_1 = \frac{p_3}{\rho} + \frac{V_3^2}{2\alpha} + gz_3 + \Sigma F_{\text{exp}}$$

or

$$\Sigma F_{\text{exp}} = \frac{p_1 - p_3}{\rho} + (z_1 - z_3)g + \frac{V_1^2 - V_3^2}{2\alpha} \tag{3.94}$$

For a horizontal system, $z_1 = z_3$, putting $p_1 \approx p_2$ and substituting for α from equation (3.16), equations (3.93) and (3.94) yield the following expression for the head loss, h_e:

$$h_e = \frac{\Sigma F_{\text{exp}}}{g} = \frac{1}{g}\left(\frac{Q}{A_1}\right)^2\left(\frac{3n+1}{2n+1}\right) \times \left[\frac{(n+3)}{2(5n+3)}\left(\frac{A_1}{A_2}\right)^2 - \left(\frac{A_1}{A_2}\right) + \frac{3(3n+1)}{2(5n+3)}\right] \tag{3.95}$$

If n were equal to zero, the velocity would be uniform across the pipe cross-section ($\alpha = 1$) and equation (3.95) would reduce to

$$h_e = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 \tag{3.96}$$

This agrees with the expression for turbulent Newtonian flow when the velocity profile is assumed to be approximately flat.

3.8.2 Entrance effects for flow in tubes

The previous discussion on flow in pipes has been restricted to fully-developed flow where the velocity at any position in the cross-section is independent of distance along the pipe. In the entrance and exit sections of the pipe this will no longer be true. Since exit effects are much less significant than entrance effects, only the latter are dealt with in detail here.

For all fluids entering a small pipe from either a very much larger one or from a reservoir, the initial velocity profile will be approximately flat, and will then undergo a progressive change until fully developed flow is established, as shown schematically in Figure 3.24.

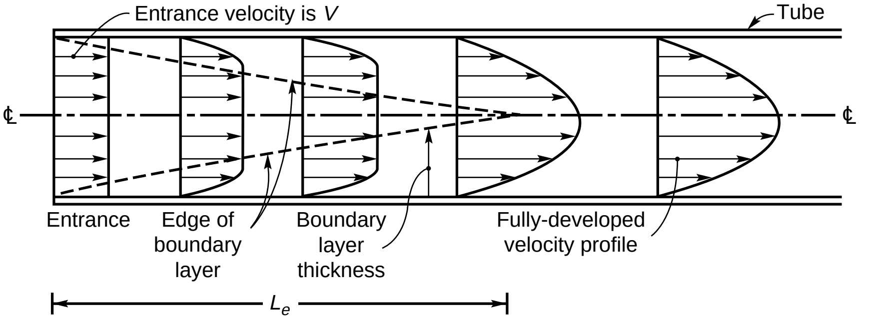

Figure 3.24 *Development of the boundary layer and velocity profile for laminar flow in the entrance region of a pipe*

The thickness of the boundary layer is theoretically zero at the entrance and increases progressively along the tube. The retardation of the fluid in the wall region must be accompanied by a concomitant acceleration in the central region in order to maintain continuity. When the velocity profile has reached its final shape, the flow is fully developed and the boundary layers may be considered to have converged at the centre line. It is customary to define an entry length, L_e, as the distance from the inlet at which the centreline velocity is 99% of that for the fully-developed flow. The pressure gradient in this entry region is different from that for fully developed flow and is a function of the initial velocity profile. There are two factors influencing the pressure gradient in the entry region: firstly, some pressure energy is converted into kinetic energy as the fluid in the central core accelerates, and secondly, the higher velocity gradients in the wall region result in greater frictional losses. It is important to estimate both the pressure drop occurring in the region before flow has been fully developed and the extent of this entrance length. This situation is amenable to analysis by repeated use of the mechanical energy balance equation. Consider the schematics of the flow shown in Figure 3.25. The stations '1' and '3' are well removed from the tube entrance, '2' is in the plane of entrance while '3' is situated in the fully developed region. The frictional pressure loss between points '1' and '3' can be expressed as:

$$\rho\Sigma F = \Delta p_{fd(1-2)} + \Delta p_{fd(2-3)} + \Delta p_{ex(1-2)} + \Delta p_{ex(2-3)} \quad (3.97)$$

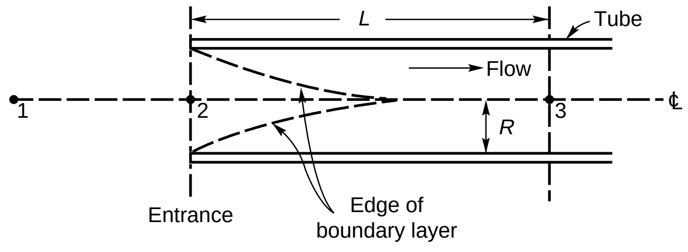

Figure 3.25 *Schematics for calculation of entrance effects*

where the subscripts 'fd' and 'ex' respectively denote the pressure drops over the regions of fully-developed flow and the additional pressure drop due to the acceleration of the fluid. Because $V_1 \ll V_3$, the fully developed pressure loss between points '1' and '2' is assumed to be negligible and that between '2' and '3' can be expressed in terms of the wall shear stress in the smaller tube as:

$$\Delta p_{fd(2-3)} = 2\tau_w \left(\frac{L}{R}\right. \quad (3.98)$$

where L is the length of the pipe between '2' and '3'. Thus, the extra frictional loss between '1' and '3' arising from the fact that flow is developing, $\Delta p_{\text{entrance}}$, can be written as:

$$\Delta p_{\text{entrance}} = (p_1 - p_3) - \Delta p_{fd(2-3)} = p_1 - p_3 - 2\tau_w \left(\frac{L}{R}\right. \quad (3.99)$$

Applying the mechanical energy balance between points '1' and '3', noting $z_1 = z_3$ and $V_1 \ll V_3$, and substituting for $\rho\Sigma F$ from equations (3.97) and (3.98):

$$(p_1 - p_3) = \frac{\rho V_3^2}{2\alpha_3} + \rho\Sigma F = \frac{\rho V_3^2}{2\alpha_3} + 2\tau_w \left(\frac{L}{R}\right. \quad + \Delta p_{ex(1-3)} + \Delta p_{ex(2-3)}$$

or

$$\Delta p_{\text{entrance}} = p_1 - p_3 - 2\tau_w \left(\frac{L}{R} \right. = \frac{\rho V_3^2}{2\alpha_3} + \Delta p_{ex(1-2)} + \Delta p_{ex(2-3)} \tag{3.100}$$

Noting $(1/2)\rho V_3^3 = \tau_w/f$ and that $f = 16/\text{Re}_{MR}$ in laminar region, it is customary to re-arrange equation (3.100) as:

$$\frac{\Delta p_{\text{ent}}}{2\tau_w} = C_1(n)\frac{\text{Re}_{MR}}{32} + C_2(n) \tag{3.101}$$

$$\text{where} \quad C_1(n) = \left(\frac{1}{\alpha_3} + \frac{\Delta p_{ex(2-3)}}{(1/2)\rho V_3^2} \right. \tag{3.102}$$

$$\text{and} \quad C_2(n) = \frac{\Delta p_{ex(1-2)}}{2\tau_w} \tag{3.103}$$

For laminar flow of power-law fluids, it has been found that both C_1 and C_2 (also known as Couette correction) are functions of n alone. Obviously, C_2 representing the loss between points '1' and '2' would be strongly dependent on the geometrical details of the system, more gradual or smooth the entrance, smaller will be the value of C_2. However, to date, its values have been computed only for an abrupt change. Table 3.4 summarises the predicted values of C_1 and C_2 for a range of values of n. It should be noted that $C_1(n)$ decreases with the increasing degree of shear-thinning behaviour while $C_2(n)$ shows the exactly opposite type of dependence on n. That is, the contribution of the excess pressure drop between points '1' and '2' increases with decreasing value of n. Based on extensive comparisons

Table 3.4 *Values of $C_1(n)$ and $C_2(n)$ [Boger, 1987]*

n	$C_1(n)$	$C_2(n)$
1	2.33	0.58
0.9	2.25	0.64
0.8	2.17	0.70
0.7	2.08	0.79
0.6	1.97	0.89
0.5	1.85	0.99
0.4	1.70	1.15
0.3	1.53	1.33

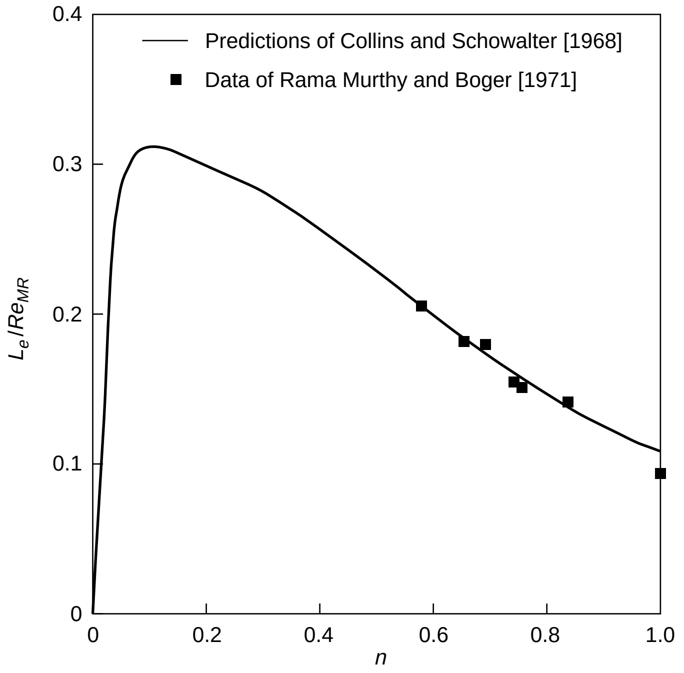

Figure 3.26 *Entrance length for power-law fluids*

between predictions and experimental data, this approach has been found to be reliable for estimating the value of Δp_{en} for linear contraction ratios greater than 2 and downstream Reynolds number $(\rho V^{2-n} D^n/m) > 5$ [Boger, 1987]. Figure 3.26 shows the entry length L_e, required to attain fully developed flow in tubes; excellent agreement is seen to exist between predictions and limited experimental data. From a practical standpoint, the currently available body of information suggests that the entrance length, L_e, is of the order of forty pipe diameters for inelastic fluids and about 110D for visco-elastic fluids [Cho and Hartnett, 1982] in streamline flow. The literature on this subject has been critically reviewed by Boger [1987].

There is little information on either the entrance length or the additional pressure drop for fully developed turbulent flow. Dodge and Metzner [1959] indicated that both the entrance length and the extra pressure loss for inelastic fluids were similar to those for Newtonian fluids.