5.2.2 Drag on a sphere in viscoplastic fluids

By virtue of its yield stress, a viscoplastic material in an unsheared state will support an immersed particle for an indefinite period of time. In recent years, this property has been successfully exploited in the design of slurry pipelines, as briefly discussed in section 4.3. Before undertaking an examination of the drag force on a spherical particle in a viscoplastic medium, the question of static equilibrium will be discussed and a criterion will be developed to delineate the conditions under which a sphere will either settle or be held stationary in a liquid exhibiting a yield stress.

(i) Static equilibrium

The question of whether or not a sphere will settle in an unsheared viscoplastic material has received considerable attention in the literature [Chhabra and Uhlherr, 1988; Chhabra, 1993a]. For the usual case where the sphere is acted upon by gravity, it is convenient to introduce a dimensionless group, Y, which denotes the ratio of the forces due to the yield stress and due to gravity. Neglecting numerical constants, the simplest definition of Y is

$$Y = \frac{\tau_0}{gd(\rho_s - \rho)} \tag{5.6}$$

Thus, small values of Y will favour motion of a sphere. The critical values of Y reported by various investigators [Chhabra and Uhlherr, 1988] fall in two categories. One group, with the value of Y in the range 0.06 ± 0.02, includes the numerical predictions [Beris *et al*. 1985], observations on the motion/no motion of spheres under free fall conditions [Ansley and Smith, 1967] and the residual force upon the cessation of flow [Brookes and Whitmore, 1968]. The second group, with $Y \sim 0.2$, relies on the intuitive consideration that the buoyant weight of the sphere is supported by the vertical component of the force due to the yield stress, and on measurements on a fixed sphere held in an unsheared viscoplastic material [Uhlherr, 1986]. The large discrepancy between the two sets of values suggests that there is a fundamental difference in the underlying mechanisms inherent in these two approaches. Additional complications arise from the fact that the values of yield stress (τ_0) obtained using different methods differ widely [Nguyen and Boger, 1992]. Thus, it is perhaps best to establish the upper and lower bounds on the size and/or density of a sphere that will settle in particular circumstances.

(ii) Flow field

As in the case of the solid plug-like motion of viscoplastic materials in pipes and slits (discussed in Chapter 3), there again exists a bounded zone of flow associated with a sphere moving in a viscoplastic medium, and beyond this zone, the fluid experiences elastic deformation, similar to that in elastic solids [Volarovich and Gutkin, 1953; Tyabin, 1953]. Indeed the difficulty in delineating the interface between the flow and no flow zones has been the main impediment to obtaining numerical solutions to this problem. Furthermore, even within the cavity of shear deformation, there is unsheared material adhering to parts of the sphere surface and this suggests that the yield stress may not act over the entire surface. The existence of such unsheared material attached to a moving sphere has been observed experimentally, but it is rather difficult to estimate its exact shape and size [Valentik and Whitmore, 1965; Atapattu *et al*., 1995].

Using the laser speckle photographic method, Atapattu *et al*. [1995] measured point velocities in the fluid near a sphere moving at a constant speed

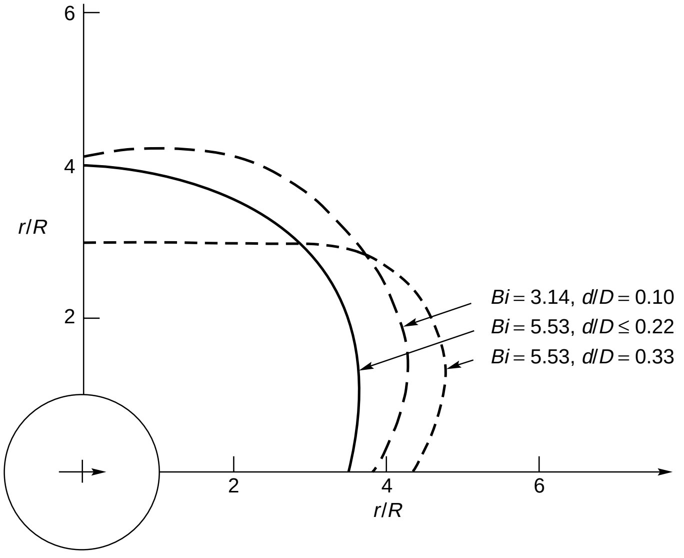

Figure 5.3 *Size of sheared cavity around a sphere moving in a viscoplastic (aqueous carbopol) solution*

on the axis of a cylindrical tube containing viscoplastic carbopol solutions. Notwithstanding the additional effects arising from the walls of the tube, Figure 5.3 shows the typical size and shape of deformation cavity for a range of values of the sphere to tube diameter ratio and the Bingham number, Bi $(= \tau_0^B d/V\mu_B)$. The slight difference between the size of cavity in the radial and axial directions should be noted, especially for large values of sphere to tube diameter ratio (d/D), but the deformation envelope rarely extends beyond 4–5 sphere radii. Nor has it been possible to identify small caps of solid regions near the front and rear stagnation points.

Example 5.2

A china clay suspension has a density of 1050 kg/m^3 and a yield stress of 13 Pa. Determine the diameter of the smallest steel ball (density 7750 kg/m^3) which will settle under its own weight in this suspension.

Solution

Here $\rho = 1050\,\text{kg/m}^3$; $\rho_s = 7750\,\text{kg/m}^3$

$\tau_0 = 13\,\text{Pa}$

From equation (5.6), the sphere will settle only if $Y < {\sim}0.04 - 0.05$

Substituting values, $$\frac{\tau_0}{gd(\rho_s - \rho)} = \frac{13}{9.81 \times d \times (7750 - 1050)} \leq 0.04$$

or $$d = 4.9\,\text{mm}.$$

For a less conservative estimation, $Y = 0.212$ may be used. The use of this criterion gives $d = 0.93\,\text{mm}$. Thus, a 5 mm sphere will definitely settle in this suspension, but there is an element of uncertainty about the 1 mm steel ball.

(iii) Drag force

The main difficulty in making theoretical estimates of the drag force on a sphere moving in a viscoplastic medium has been the lack of quantitative information about the shape of the sheared cavity. Both Beris *et al.* [1985] and Blackery and Mitsoulis [1997] have used the finite element method to evaluate the total drag on a sphere moving slowly (creeping regime) in a Bingham plastic medium and have reported their predictions in terms of the correction factor, X, $(= C_D \text{Re}_B/24)$ which now becomes a function of the Bingham number, Bi $(= \tau_0^B d/V\mu_B)$ as:

$$X = 1 + a(\text{Bi})^b \tag{5.7}$$

While Beris *et al.* [1985] evaluated the drag in the absence of walls (i.e. $d/D = 0$), Blackery and Mitsoulis [1997] have numerically computed the value of X for a range of diameter ratios $0 \leq d/D \leq 0.5$ and up to $\text{Bi} = 1000$. For the case of $(d/D) = 0$ (i.e. no wall effects), $a = 2.93$ and $b = 0.83$. In the range $0 \leq (d/D) \leq 0.5$, the values of a and b vary monotonically in the ranges $1.63 \leq a \leq 2.93$ and $0.83 \leq b \leq 0.95$, respectively. As the Bingham number progressively becomes smaller, X would be expected to approach unity. The higher drag $(X > 1)$ in a viscoplastic medium is attributable to the additive effects of viscosity and yield stress.

In addition, many workers have reported experimental correlations of their drag data for spheres falling freely or being towed in viscoplastic media [Chhabra and Uhlherr, 1988; Chhabra, 1993a; Atapattu *et al.*, 1995]; most correlations are based on the use of the Bingham model, though some have found the three parameter Herschel–Bulkley fluid model (equation 1.17) to correlate their data somewhat better [Sen, 1984; Atapattu *et al.*, 1995; Beaulne and Mitsoulis, 1997]. At the outset, it is important to establish the criterion for creeping flow in viscoplastic fluids. For a sphere falling in a Newtonian fluid $(\tau_0 = 0)$, the creeping flow is assumed to occur up to about $\text{Re} \sim 1$. One of the characteristics of creeping flow in a Newtonian fluid is the reciprocal relationship between the Reynolds number and drag coefficient, i.e. $C_D\text{Re} = 24$. For Bingham plastic fluids, intuitively this product must be a function of the Bingham number, as can be seen in equation (5.7). Applying this criterion to the available data, the maximum value of the Reynolds number, Re ($= \rho V d/\mu_B$), for creeping flow is given as [Chhabra and Uhlherr, 1988]:

$$\mathrm{Re}_{\max} \simeq 100\,\mathrm{Bi}^{0.4} \tag{5.8}$$

Thus, the greater the Bingham number, the higher is the Reynolds number up to which the creeping flow conditions apply for spheres moving in Bingham plastic fluids.

As mentioned previously, the three parameter Herschel–Bulkley fluid model gives a somewhat better fit of the fluid rheology than the Bingham model. Atapattu *et al*. [1995] put forward the following semi-empirical correlation for drag on spheres in Herschel–Bulkley model liquids:

$$C_D = \frac{24}{\mathrm{Re}}(1 + \mathrm{Bi}^*) \tag{5.9}$$

where the Reynolds number, $\mathrm{Re} = \rho V^{2-n} d^n/m$ and the modified Bingham number, $\mathrm{Bi}^* = \tau_0^H/m(V/d)^n$. Equation (5.9) covers the ranges: $10^{-5} \le \mathrm{Re} \le 0.36$; $0.25 \le \mathrm{Bi}^* \le 280$; and $0.43 \le n \le 0.84$; and it also correlates the scant literature data available in the creeping flow region [Sen, 1984; Hariharaputhiran *et al*., 1998]. These results are also in line with the numerical predictions for Herschel–Bulkley fluids [Beaulne and Mitsoulis, 1997].

In the intermediate Reynolds number region, though some predictive expressions have been developed, e.g. see Chhabra [1993a] but most of these data are equally well in line with the standard drag curve for Newtonian liquids [Machac *et al*., 1995].

Thus, in summary, the non-Newtonian characteristics seem to be much more important at low Reynolds numbers and their role progressively diminishes as the inertial effects become significant with the increasing Reynolds number. Therefore, in creeping flow region, equations (5.4), (5.7) and (5.9), respectively, should be used to estimate drag forces on spheres moving in power-law and Bingham model or Herschel–Bulkley fluids. On the other hand, at high Reynolds number, the application of the standard drag curve for Newtonian fluids yields values of drag on spheres which are about as accurate as the empirical correlations available in the literature. The Reynolds number defined as $\rho V^{2-n} d^n/m$ for power-law fluids, as $\rho V d/\mu_B$ for Bingham plastics and as $(\rho V^{2-n} d^n/m)/(1 + \mathrm{Bi}^*)$ for Herschel–Bulkley model fluids must be used in the standard Newtonian drag curve.

5.2.3 Drag in visco-elastic fluids

From a theoretical standpoint, the creeping-flow steady translation motion of a sphere in a visco-elastic medium has been selected as one of the benchmark problems for the validation of procedures for numerical solutions [Walters and Tanner, 1992; Chhabra, 1993a]. Unfortunately, the picture which emerges is not only incoherent but also inconclusive. Most simulation studies are based on the creeping flow assumption (zero Reynolds number) and take into account the influence of fluid visco-elasticity on the drag of a sphere in the absence of shear-thinning behaviour. Early studies suggested a slight reduction (∼5–10%) in drag below the Stokes value, with the amount of drag reduction showing a weak dependence on Deborah or Weissenberg number (defined as $\lambda_f V/d$). However, more recent simulations [Degand and Walters, 1995] suggest that after an initial period of reduction, the drag on a sphere in a visco-elastic medium can exceed that in a Newtonian medium at high values of Deborah number; the latter enhancement is attributed to extensional effects of the fluid. Both drag reduction (up to 25%) and enhancements (up to 200%) compared with the Newtonian value have been observed experimentally [Chhabra, 1993a]. However there is very little quantitative agreement among various workers between the results of numerical simulations and experimental studies. The former seem to be strongly dependent on the details of the numerical procedure, mesh size, etc, while the experimental results appear to be very sensitive to the chemical nature, water purity, etc. of the polymer solutions used. It is not yet possible to interpret and/or correlate experimental results of drag in visco-elastic fluids in terms of measureable rheological properties. Aside from these uncertainties, other time-dependent effects have also been observed. For instance, unlike the monotonic approach to the terminal velocity in Newtonian and power-law type fluids [Bagchi and Chhabra, 1991; Chhabra *et al*., 1998], a sphere released in a visco-elastic liquid could attain a transitory velocity almost twice that of its ultimate falling velocity [Walters and Tanner, 1992].

On the other hand, the effects of shear-thinning viscosity completely overshadow those of visco-elasticity, at least in the creeping flow region. Indeed, a correlation based on a viscosity model, with zero shear viscosity and/or a characteristic time constant, provides satisfactory representation of drag data when the liquid exhibits both shear-thinning properties and visco-elasticity [Chhabra, 1993a].