3.4 Friction factors for transitional and turbulent conditions

Though turbulent flow conditions are encountered less frequently with polymeric non-Newtonian substances, sewage sludges, coal and china clay suspensions are usually all transported in the turbulent flow regime in large diameter pipes. Therefore, considerable research efforts have been directed at developing a generalised approach for the prediction of the frictional pressure drop for turbulent flow in pipes, especially for purely viscous (power-law, Bingham plastic and Herschel–Bulkley models) fluids. Analogous studies for the flow of visco-elastic and the so-called drag-reducing fluids are somewhat inconclusive. Furthermore, the results obtained with drag-reducing polymer solutions also tend to be strongly dependent on the type and molecular weight of the polymers, the nature of the solvent and on the type of experimental set up used, and it is thus not yet possible to put forward generalised equations for the turbulent flow of such fluids. Therefore, the ensuing discussion is restricted primarily to the turbulent flow of time-independent fluids. However, excellent survey articles on the turbulent flow of visco-elastic and drag-reducing systems are available in the literature [Govier and Aziz, 1982; Cho and Hartnett, 1982; Sellin *et al*., 1982].

In the same way as there are many equations for predicting friction factor for turbulent Newtonian flow, there are numerous equations for time-independent non-Newtonian fluids; most of these are based on dimensional considerations combined with experimental observations [Govier and Aziz, 1982; Heywood and Cheng, 1984]. There is a preponderance of correlations based on the power-law fluid behaviour and additionally some expressions are available for Bingham plastic fluids [Tomita, 1959; Wilson and Thomas, 1985]. Here only a selection of widely used and proven methods is presented.

3.4.1 Power-law fluids

In a comprehensive study, Dodge and Metzner [1959] carried out a semi-empirical analysis of the fully developed turbulent flow of power-law fluids in smooth pipes. They used the same dimensional considerations for such fluids, as Millikan [1939] for incompressible Newtonian fluids, and obtained an expression which can be re-arranged in terms of the apparent power law index, n', (equation 3.26) as follows:

$$\frac{1}{\sqrt{f}} = A(n')\log[\mathrm{Re}_{MR} f^{(2-n')/2}] + C(n') \tag{3.35}$$

where $A(n')$ and $C(n')$ are two unknown functions of n'. Based on extensive experimental results in the range $2900 \le \mathrm{Re}_{MR} \le 36\,000$; $0.36 \le n' \le 1$ for polymer solutions and particulate suspensions, Dodge and Metzner [1959] obtained,

$$A(n') = 4(n')^{-0.75} \tag{3.36a}$$

$$C(n') = -0.4(n')^{-1.2} \tag{3.36b}$$

Incorporating these values in equation (3.35),

$$\frac{1}{\sqrt{f}} = \frac{4}{(n')^{0.75}}\log[\mathrm{Re}_{MR} f^{(2-n')/2}] - \frac{0.4}{(n')^{1.2}} \tag{3.37}$$

and this relation is shown graphically in Figure 3.6.

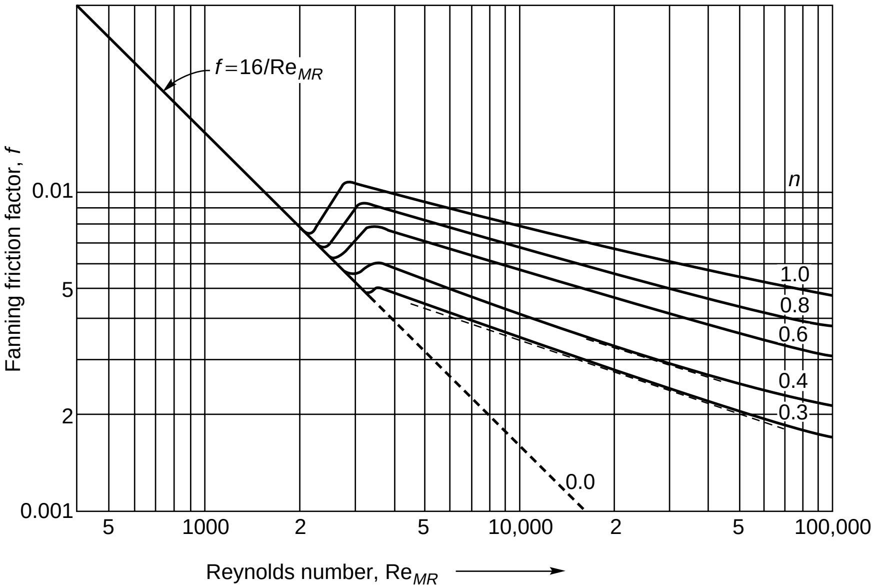

Figure 3.6 *Friction factor – Reynolds number behaviour for time-independent fluids [Dodge and Metzner, 1959]*

A more detailed derivation of equation (3.37) is available in their original paper and elsewhere [Skelland, 1967]. For Newtonian fluids ($n' = 1$), equation (3.37) reduces to the well-known Nikuradse equation. Dodge and Metzner [1959] also demonstrated that their data for clay suspensions which did not conform to power-law behaviour, were consistent with equation (3.37) provided that the slope of $\log \tau_w - \log(8V/D)$ plots was evaluated at the appropriate value of the wall shear stress. It is also important to point out here that equation (3.37) necessitates the values of n' and m' be evaluated from volumetric flow rate – pressure drop data for laminar flow conditions. Often, this requirement poses significant experimental difficulties. Finally, needless to say, this correlation is implicit in friction factor f (like the equation for Newtonian fluids) and hence an iterative technique is needed for its solution. The recent method of Irvine [1988] obviates this difficulty. Based on the Blasius expression for velocity profile for turbulent flow (discussed subsequently) together with modifications based on experimental results, Irvine [1988] proposed the following Blasius like expression for power-law fluids:

$$f = \{\mathrm{D}(n)/\mathrm{Re}_{MR}\}^{1/(3n+1)} \tag{3.38}$$

where $\mathrm{D}(n) = \dfrac{2^{n+4}}{7^{7n}}\left(\dfrac{4n}{3n+1}\right)^{3n^2}$

Note that this cumbersome expression does reduce to the familiar Blasius expression for $n = 1$ and is explicit in friction factor, f. Equation (3.38) was stated to predict the values of friction factor with an average error of ±8% in the range of conditions: $0.35 \le n \le 0.89$ and $2000 \le \mathrm{Re}_{MR} \le 50\,000$. Though this approach has been quite successful in correlating most of the literature data, significant deviations from it have also been observed [Harris, 1968; Quader and Wilkinson, 1980; Heywood and Cheng, 1984]; though the reasons for such deviations are not immediately obvious but possible visco-elastic effects and erroneous values of the rheological parameters (e.g. n' and m') cannot be ruled out. Example 3.6 illustrates the application of these methods.

Example 3.6

A non-Newtonian polymer solution (density 1000 kg/m^3) is in steady flow through a smooth 300 mm inside diameter 50 m long pipe at the mass flow rate of 300 kg/s. The following data have been obtained for the rheological behaviour of the solution using a tube viscometer. Two tubes, 4 mm and 6.35 mm in inside diameter and 2 m and 3.2 m long respectively were used to encompass a wide range of shear stress and shear rate.

Mass flow rate (kg/h)	Pressure drop (kPa)	Mass flow rate (kg/h)	Pressure drop (kPa)
$D = 4$ mm, $L = 2$ m		$D = 6.35$ mm, $L = 3.2$ m	
33.9	49	18.1	27
56.5	57.6	45.4	36
95	68.4	90.7	44
136	76.8	181	54
153.5	79.5	272	61

Determine the pump power required for this pipeline. How will the power requirement change if the flow rate is increased by 20%?

Solution

First, the tube viscometer data will be converted to give the wall shear stress, τ_w, and nominal shear rate, $(8V/D)$:

$$\tau_w = \frac{D}{4}\left(\frac{-\Delta p}{L}\right) = \frac{4\times 10^{-3}}{4} \times \frac{49\times 1000}{2} = 24.5\ \mathrm{Pa}$$

and $\dfrac{8V}{D} = \dfrac{8}{4 \times 10^{-3}} \times \dfrac{33.9}{1000 \times 3600} \times \dfrac{4}{\pi(4 \times 10^{-3})^2} = 1499\,\text{s}^{-1}$

Similarly the other mass flow rate–pressure drop data can be converted into $\tau_w - (8V/D)$ form as are shown in the table.

τ_w (Pa)	$(8V/D)$ (s^{-1})	τ_w (Pa)	$(8V/D)$ (s^{-1})
$D = 4\,\text{mm}$		$D = 6.35\,\text{mm}$	
24.5	1499	13.4	200
28.8	2500	17.86	502
34.2	4200	21.83	1002
38.4	6000	26.8	2000
39.8	6800	30.26	3005

Note that since L/D for both tubes is 500, entrance effects are expected to be negligible. Figure 3.7 shows the $\tau_w - (8V/D)$ data on log-log coordinates. Obviously, n' is not constant, though there seem to be two distinct power-law regions with parameters:

$$n' = 0.3 \quad m' = 2.74\,\text{Pa}\cdot\text{s}^{n'} \quad (\tau_w < \sim 30\,\text{Pa})$$

$$n' = 0.35 \quad m' = 1.82\,\text{Pa}\cdot\text{s}^{n'} \quad (\tau_w > \sim 30\,\text{Pa})$$

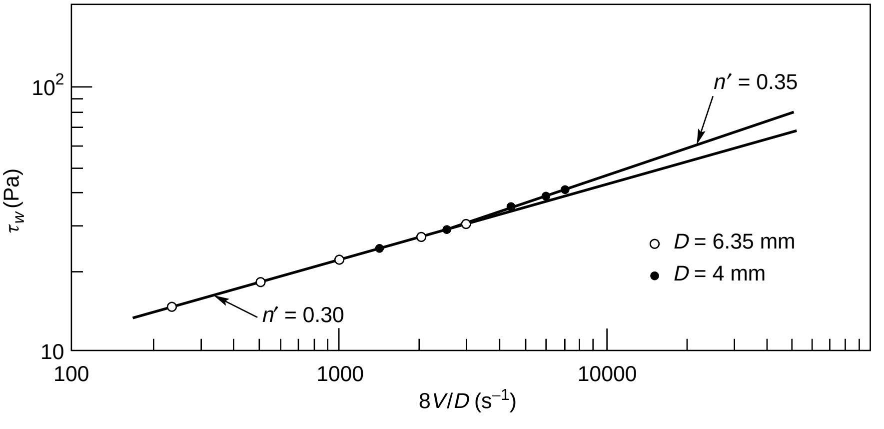

Figure 3.7 *Wall shear stress–apparent wall shear rate plot for data in example 3.6*

Also, the overlap in data obtained using tubes of two different diameters confirms the time-independent behaviour of the solution.

In the large pipe, the mean velocity of flow, $V = \dfrac{300}{1000} \times \dfrac{4}{\pi(0.3)^2}$

or $V = 4.1\,\text{m/s}$

Let us calculate the critical velocity, V_c, for the end of the streamline flow by setting $\mathrm{Re}_{MR} = 2100$, i.e.

$$\frac{\rho V^{2-n'} D^{n'}}{8^{n'-1} m'} = 2100$$

Substituting values,

$$\frac{1000 \times V_c^{2-0.3} \times (0.3)^{0.3}}{8^{0.3-1} \times 2.74} = 2100$$

Solving, $V_c = 1.47$ m/s which is lower than the actual velocity of 4.1 m/s and hence, the flow in the 300 mm pipe is likely to be turbulent.

Initially, let us assume that the wall shear stress in the large pipe would be <30 Pa, i.e. $n' = 0.3$ and $m' = 2.74\ \mathrm{Pa{\cdot}s}^n$ can be used for calculating the value of the Re_{MR}, equation (3.28b),

$$\mathrm{Re}_{MR} = \frac{\rho V^{2-n'} D^{n'}}{8^{n'-1} m'} = \frac{1000 \times 4.1^{2-0.3} \times (0.3)^{0.3}}{8^{0.3-1} \times 2.74}$$
$$= 12\,230$$

Now for $n' = 0.3$ and $\mathrm{Re}_{MR} = 12\,230$, from Figure 3.6, $f \approx 0.0033$ (equation (3.38) gives $f = 0.0036$). The frictional pressure gradient $(-\Delta p/L)$, is calculated next:

$$\left(\frac{-\Delta p}{L}\right) = \frac{2 f \rho V^2}{D} = \frac{2 \times 0.0033 \times 1000 \times 4.1^2}{0.3} = 364\ \mathrm{Pa/m}$$

The value of $\tau_w = (D/4)(-\Delta p/L) = (0.3 \times 364)/4 = 27.7$ Pa is within the range of the first power-law region and hence no further iteration is needed. The pump power is $Q \cdot \Delta p$, i.e. $(300/1000) \times 364 \times 50 = 5460$ W.

For the case when the flow rate has been increased by 20%, i.e. the new mass flow rate in the large pipe is 360 kg/s.

$$\text{mean velocity of flow, } V = \frac{360}{1000} \times \frac{4}{\pi (0.3)^2} = 4.92\ \mathrm{m/s}$$

Based on the previous calculation, it is reasonable to assume that the new value of the wall shear stress would be greater than 30 Pa and therefore, one should use $n' = 0.35$ and $m' = 1.82\ \mathrm{Pa{\cdot}s}^{n'}$.

$$\text{The Reynolds number, } \mathrm{Re}_{MR} = \frac{1000 \times 4.92^{2-0.35} \times (0.3)^{0.35}}{8^{0.35-1} \times 1.82}$$
$$= 19\,410$$

For $n' = 0.35$ and $\mathrm{Re}_{MR} = 19\,410$, from Figure 3.6, $f \approx 0.0032$ (while equation (3.38) also yields the same value). The frictional pressure gradient $(-\Delta p/L)$ is:

$$\frac{-\Delta p}{L} = \frac{2 f \rho V^2}{D} = \frac{2 \times 0.0032 \times 1000 \times 4.92^2}{(0.3)} = 511\ \mathrm{Pa/m}$$

Checking: $\tau_w = \frac{D}{4}\left(\frac{-\Delta p}{L}\right) = \frac{0.3 \times 511}{4} = 39\,\text{Pa}$.

This value is just within the range of laminar flow data.

$$\text{pump power} = \frac{360}{1000} \times 511 \times 50 = 9200\,\text{W}$$