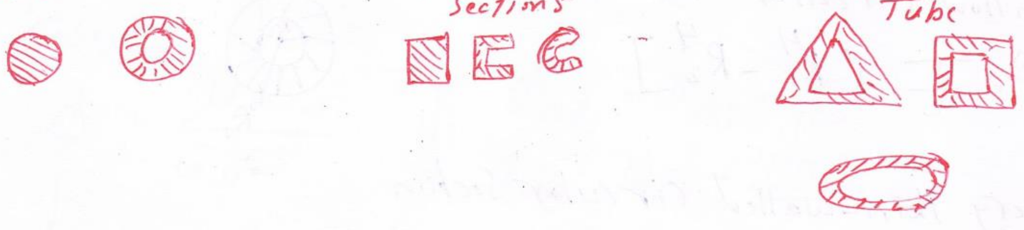
Torsion اللـي المحاضرة السابعة

I) Torsion of circular Section

II) Torsion of non-circular Sections

III) Torsion of non-circular thin-walled Tube



I) Torsion of circular Sections:-

$$\tau_{\rho} = \frac{M_t R}{J}$$ Torsional formula

اجهاد القص عند اي نقطة تبعد مسافة ρ من مركز المقطع

τ_{ρ} :- Shear stress at any point at a distance ρ from the center section

R : the distance from the center section to the point you want to calculate its J

المسافة من مركز المقطع الى النقطة المراد حسابها

M_t :- internal torsion moment — عزم اللي الداخلي

T :- external torsion moment — عزم اللي الخارجي المسلط

J :- Polar moment of inertia — عزم القصور الذاتي القطبي

$$\tau_{max} = \frac{M_t R}{J}$$

كيفية حساب J :

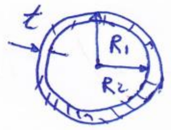
Solid circular section :

$$J = \frac{\pi}{2} R^4$$

Hollow circular section

$$J = \frac{\pi}{2} [R_1^4 - R_2^4]$$

Very thin-walled circular section



$$R_2 = R_1 - t$$

$$J = \frac{\pi}{2} [R_1^4 - (R_1 - t)^4]$$

$$J = \frac{\pi}{2} [R_1^4 - (R_1^4 - 4R_1^3 t + 6R_1^2 t^2 - 4R_1 t^3 + t^4)]$$

$$J = \frac{\pi}{2} [\cancel{R_1^4} - \cancel{R_1^4} + 4R_1^3 t]$$

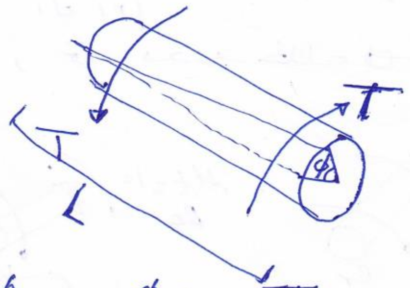
$$J = 2\pi R_1^3 t$$

if $\frac{t}{R_2} < \frac{1}{10} \Rightarrow$ very thin-walled $J = 2\pi R_1^3 t$

if $\frac{t}{R_2} \geq \frac{1}{10} \Rightarrow$ Hollow section $J = \frac{\pi}{2} [R_1^4 - R_2^4]$

Angle of Rotation (Angle of twisting) (ϕ)

توضيحي تخطيطي



Case 1

$$\phi = \frac{M_t L}{G J}$$

ϕ → rad

$$\phi_{Rad} = \phi_{deg} * \frac{\pi}{180}$$

$$\phi_{deg} = \phi_{rad} * \frac{180}{\pi}$$

يستخدم هذا القانون عندما يكون خلال الطول L العزم الداخلي M_t والمادة G والمقطع (J) ثوابت

G = Modulus of rigidity معامل الجساءة
(Modulus of elasticity in shear) (GPa) (MPa). (KPa) (Pa)

$$\boxed{\tau = G \gamma}$$ Hook's low for Shear.

τ = Shear stress

γ : Shear strain (rad)

$$G = \frac{E}{2(1+\nu)}$$

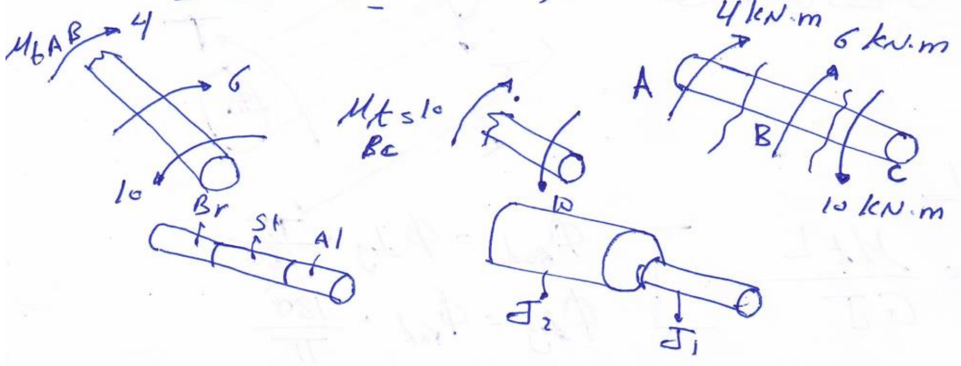
E :- modulus of elasticity

ν :- Poisson's Ratio

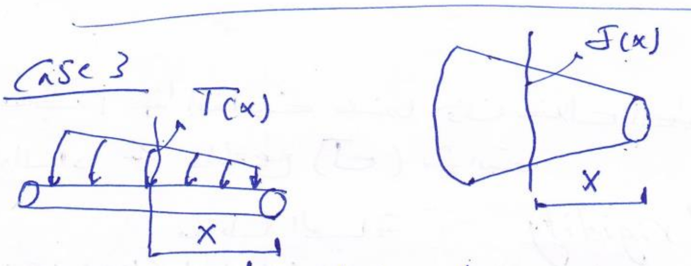
Case 2

$$\text{rad} \leftarrow \phi = \sum \frac{M_{t_i} L_i}{G_i J_i}$$

نستخدم عندما تكون M_t , J , G متغيرة خلال امتداد الطول



Case 3

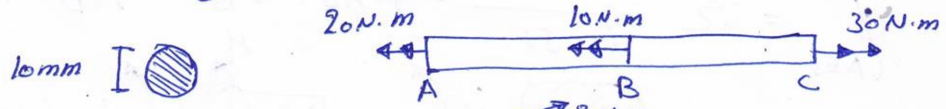


$$\phi = \int_0^L \frac{M_t(x)\, dx}{G\, J(x)}$$

نستخدم عندما تكون $M_t(x)$ و J متغيرة دالة خلال الطول L

Example

Find Max Shearing stress for shaft (AC) as shown in fig.

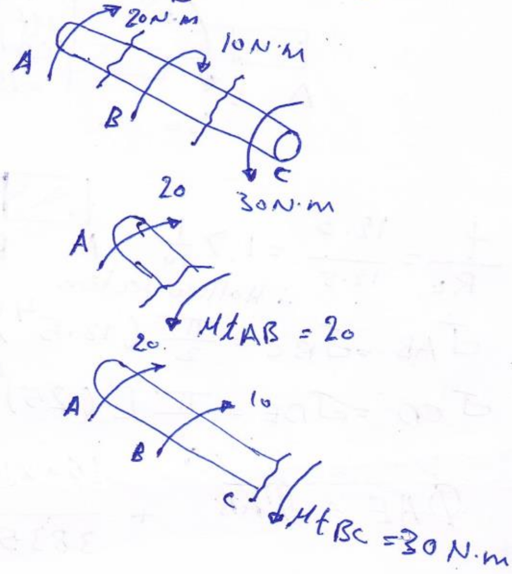


Sol

$$\tau_{max} = \frac{M_t R}{J}$$

$$\tau_{max} = \frac{30 \times 10^3 \times 5}{\frac{\pi}{2}(5)^4}$$

$$\tau_{max} = 152.8 \frac{N}{mm^2} \, (MPa)$$

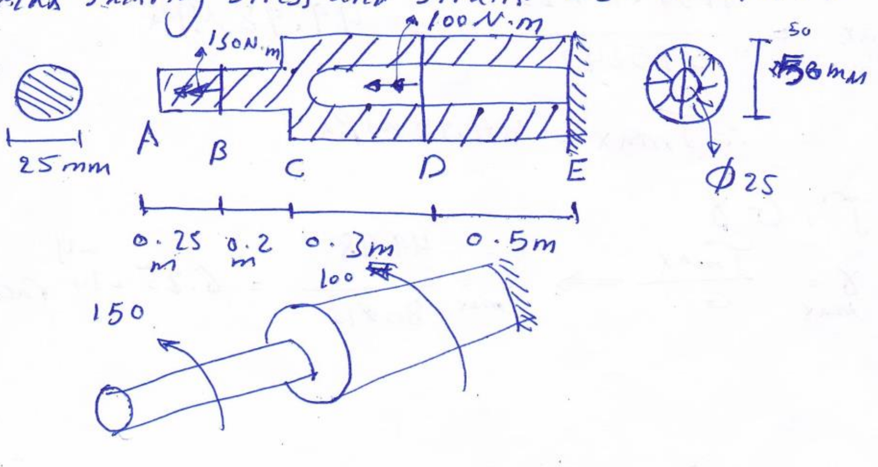


Example 1

shaft with fixed at E and free end at A is subjected to twisting moments at B and D. Determine

① Rotation of free end.

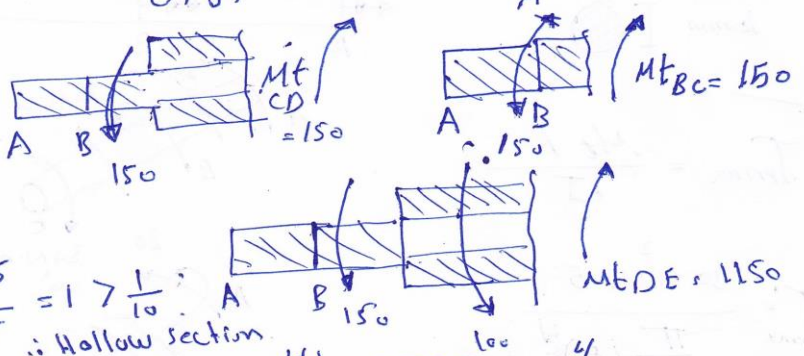
② Max shearing stress and strain. G = 80 Gpa,



Sol

بما أنه لم يحدد المقطع الثاني الذي نحسب بالنسبة لها فإننا سنعتبرها النهاية المثبتة E

$$\phi_{(AE)} = \sum \frac{Mt_i L_i}{G_i J_i} = \phi_{AB} + \phi_{BC} + \phi_{CD} + \phi_{DE}$$



$$\frac{t}{R_i} = \frac{12.5}{12.5} = 1 > \frac{1}{10}$$

$\therefore$ Hollow section

$$J_{AB} = J_{BC} = \frac{\pi}{2}(12.5^4) = 38350 \text{ mm}^4$$

$$J_{CD} = J_{DE} = \frac{\pi}{2}\left[(25)^4 - (12.5)^4\right] = 5752.43 \text{ mm}^4$$

$$\phi_{AE} = \cancel{\phi_{AB}}^{0} + \frac{150 \times 10^3 \times 200}{38350 \times 80 \times 10^3} + \frac{150 \times 10^3 \times 300}{575243 \times 80 \times 10^3}$$

$$+ \frac{1150 \times 10^3 \times 500}{575243 \times 80 \times 10^3} = 0.02325 \text{ rad}$$

$$= 1.332^\circ$$

$$\tau_{max\,(solid)} = \frac{Mt\,R}{J} = \frac{150 \times 10^3 \times 12.5}{38350} = 48.89 \text{ MPa}$$

$$\tau_{max\,(hollow)} = \frac{1150 \times 10^3 \times 25}{575243} = 49.98 \text{ MPa}$$

$$\therefore \tau_{max} = 49.98 \text{ MPa}$$

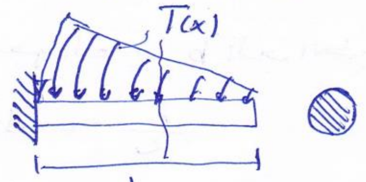
$$\tau = G\gamma$$

$$\gamma_{max} = \frac{\tau_{max}}{G} \Longrightarrow \gamma_{max} = \frac{49.98}{80 \times 10^3} = 6.25 \times 10^{-4} \text{ rad}$$

Example /

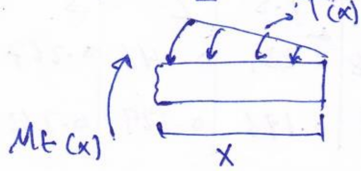
Find Max. angle of rotation for the Shaft subjected to linearly distributed torque $T(x) = kx$, GJ = Constant.

تورك موزع خطياً



sol

$$\phi_{max} = \int_0^L \frac{M_t(x)\,dx}{GJ}$$
(free end)



$\Sigma M = 0$

$$M_t(x) = \int_0^x T(x)\,dx$$

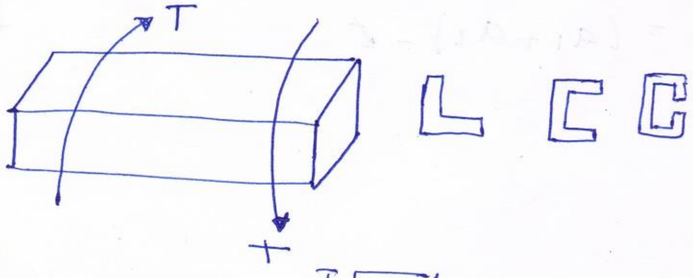
$$\int_0^x kx\,dx = \frac{kx^2}{2}\Big]_0^x = \frac{kx^2}{2}$$

$$\phi_{max} = \int_0^L \frac{\frac{kx^2}{2}\,dx}{GJ} = \frac{k}{2GJ}\left[\frac{x^3}{3}\right]_0^L$$

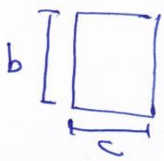
$$\boxed{\phi_{max} = \frac{kL^3}{6GJ}}$$

Part II :

Torsion of non-circular Section



$$\tau_{max} = \frac{M_t}{\alpha b c^2}$$



$$\phi = \frac{M_t L}{B b c^3 G}$$

c, b ابعاد المقطع حيث يمثل c البعد الصغير b البعد الكبير

B, α ثوابت نحصل عليها من النسبة $\frac{b}{c}$

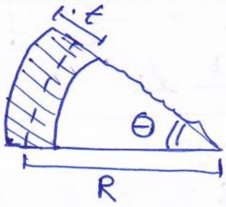
b/c	1	1.5	2	3	6	10	∞
α	0.208	0.231	0.246	0.267	0.299	0.312	0.333
B	0.141	0.196	0.229	0.263	0.299	0.312	0.333

$$\text{if } \frac{b}{c} \geq 10 \Rightarrow \alpha = B = \frac{1}{3}$$

$c = t$

$b = \theta * R$

θ → rad



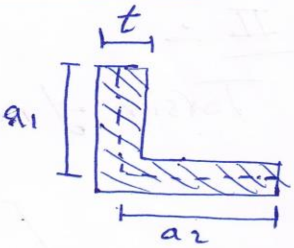
طول القوس = الزاوية المركزية المقابلة له × نصف القطر

$R * \theta = b$

$C = t$

$b = (a_1 - \frac{t}{2}) + (a_2 - \frac{t}{2})$

$= (a_1 + a_2) - t$

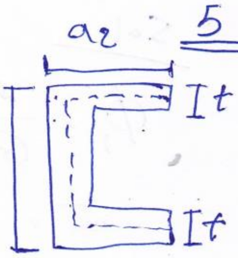


$C = t$

$b = (a_2 - \frac{t}{2}) * 2 + (a_1 - t)$

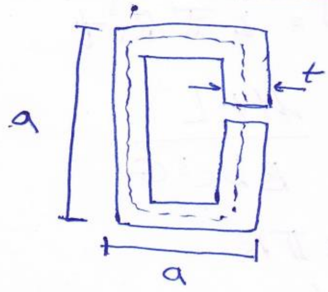
$b = (2a_2 + a_1) - 2t$

$b = (a_1 - t) + a_2 - \frac{t}{2} + a_2 - \frac{t}{2}$



$C = t$

$b = (a - t) * 4$

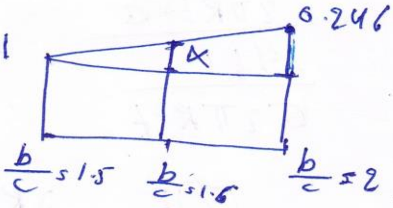


ملاحظة / اذا كانت نسبة $\frac{b}{c}$ غير موجودة في الجدول نعمل interpolation لقيم الجدول

Ex / Find α if $\frac{b}{c} = 1.6$

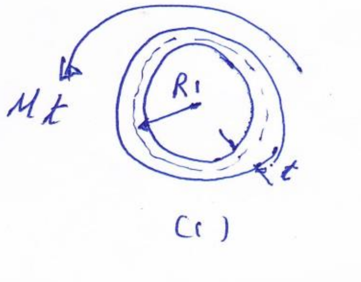
$$\frac{0.246 - 0.231}{2 - 1.5} = \frac{\alpha - 0.231}{1.6 - 1.5}$$

$\alpha = 0.234$

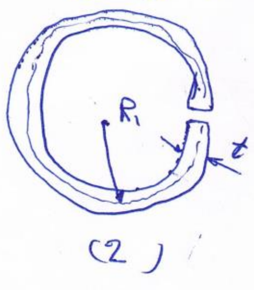


Ex Two Similar pipes of hollow circular Section are subjected to same twisting moment Mt. one of them is opened longitudinally. Find the ratio of twisting angle between them.

$\beta = \alpha = \frac{1}{3}$



(1)



(2)

Sol

$$\phi_1 = \frac{M_t L}{G J} = \frac{M_t L}{G\, 2\pi R^3 t}$$

$$J = \frac{\pi}{2}\left[R_1^4 - R_2^4\right]$$

$$= \frac{\pi}{2}\left[R_1^4 - (R_1 - t)^4\right]$$

$$J = 2\pi R_1^3 t$$

$$\phi_2 = \frac{M_t L}{B b c^3 G}$$

$b = 2\pi R$

$c = t$

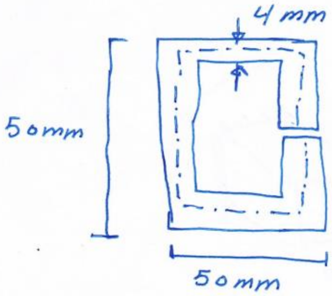
$$\phi_2 = \frac{M_t L}{\frac{1}{3} 2\pi R t^3 G} = \frac{3 M_t L}{2\pi R t^3 G}$$

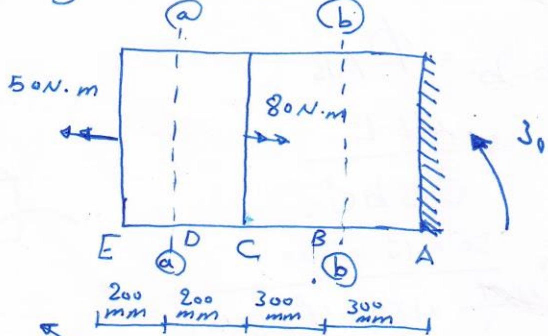
$$\frac{\phi_2}{\phi_1} = \frac{\frac{3 M_t L}{2\pi R t^3 G}}{\frac{M_t L}{G\, 2\pi R^3 t}} = \frac{3 M_t L}{2\pi R t^3 G} \times \frac{G\, 2\pi R^3 t}{M_t L}$$

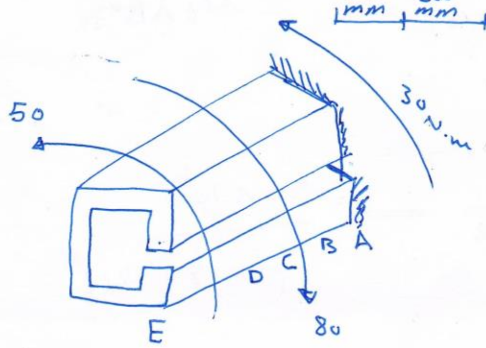
$$\frac{\phi_2}{\phi_1} = \frac{3R^2}{t^2}$$

Example

Find the angle of Rotation for Section (a-a) and (b-b) and find max shearing stress? $G = 84\,GPa$





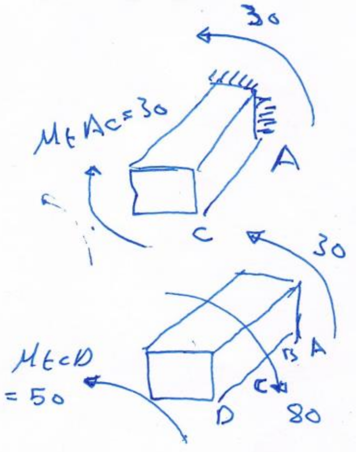


$T_A + 50 - 80$

$T_A = 30\,N.m$

Sol

$$\phi_{(a-a)} = \phi_{AC} + \phi_{CD}$$



$$b = (50-4)*4 = 184\,mm$$

$$C = 4$$

$$\frac{b}{c} = \frac{184}{4} = 46 > 10$$

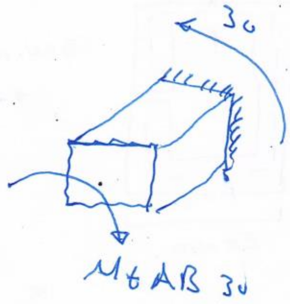
$$\alpha = \beta = \frac{1}{3}$$

$$\phi_{a-a} = \phi_{AC} + \phi_{CD}$$

$$= + \frac{30*10^3*600}{\frac{1}{3}*184(4)^3*84*10^3} - \frac{50\times10^3\times200}{\frac{1}{3}\times184\times(4)^3\times84\times10^3}$$

$$\phi_{(a-a)} = +0.02426 \text{ rad}$$

$$\phi_{b-b} = \phi_{AB}$$



$$\phi_{b-b} = \frac{M_t L}{G \beta b c^3}$$

$$= \frac{30 \times 10^3 \times 300}{\frac{1}{3} \times 184 \times 4^3 \times 84 \times 10^3}$$

$$= 0.0272 \text{ rad}$$

$$\tau_{max} = \frac{M_t}{\alpha B C^2} \Longrightarrow \frac{50 \times 10^3}{\frac{1}{3} \times 184 \times (4)^2}$$

$$\tau_{max} = 50.95 \text{ N/mm}^2 = \text{MPa}$$