Lecture 2

Force Vectors

Scalars and Vectors

Scalar. A *scalar* is any positive or negative physical quantity that can be completely specified by its *magnitude*. Examples of scalar quantities include length, mass, and time.

Vector. A *vector* is any physical quantity that requires both a *magnitude* and a *direction* for its complete description. Examples of vectors encountered in statics are force, position, and moment. A vector is shown graphically by an arrow. The length of the arrow represents the *magnitude* of the vector, and the angle θ between the vector and a fixed axis defines the *direction of its line of action*. The head or tip of the arrow indicates the *sense of direction* of the vector, Fig. 2–1.

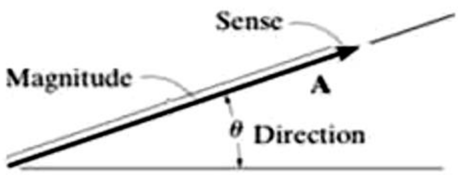


Fig. 2–1

Vector Addition. All vector quantities obey the *parallelogram law of addition*. To illustrate, the two "*component*" *vectors* **A** and **B** in Fig. 2–3*a* are added to form a "*resultant*" *vector* **R** = **A** + **B** using the following procedure:

- First join the tails of the components at a point so that it makes them concurrent, Fig. 2–3*b*.
- From the head of **B**, draw a line parallel to **A**. Draw another line from the head of **A** that is parallel to **B**. These two lines intersect at point *P* to form the adjacent sides of a parallelogram.
- The diagonal of this parallelogram that extends to *P* forms **R**, which then represents the resultant vector **R** = **A** + **B**, Fig. 2–3*c*.

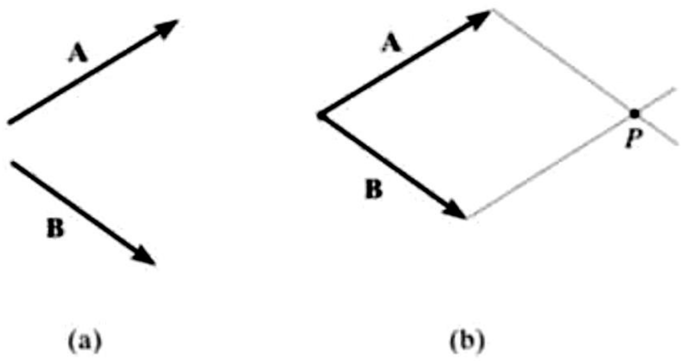


Fig. 2–3

We can also add **B** to **A**, Fig. 2–4*a*, using the *triangle rule*, which is a special case of the parallelogram law, whereby vector **B** is added to vector **A** in a "head-to-tail" fashion, i.e., by connecting the head of **A** to the tail of **B**, Fig. 2–4*b*. The resultant **R** extends from the tail of **A** to the head of **B**. In a similar manner, **R** can also be obtained by adding **A** to **B**, Fig. 2–4*c*. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e., $\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.

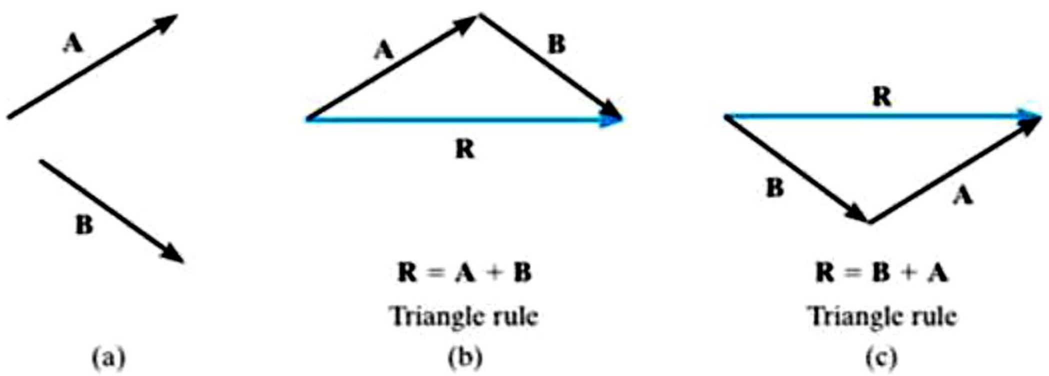


Fig. 2–4

Finding a Resultant Force. The two component forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the pin in Fig. 2–7*a* can be added together to form the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$, as shown in Fig. 2–7*b*. From this construction, or using the triangle rule, Fig. 2–7*c*, we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.

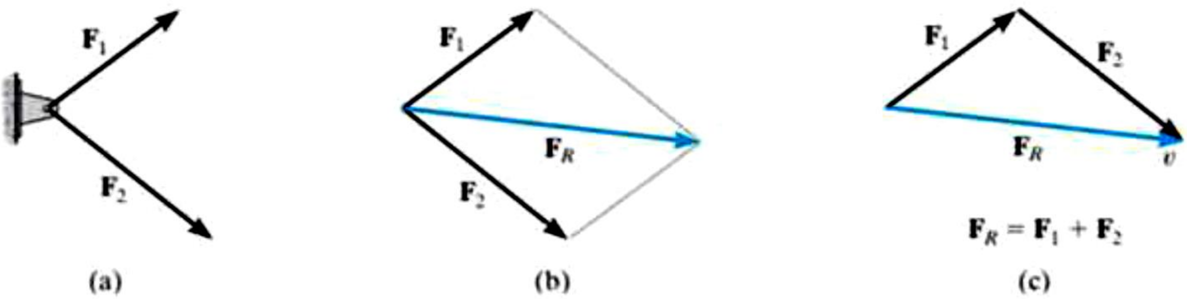


Fig. 2–7

Finding the Components of a Force. Sometimes it is necessary to resolve a force into two *components* in order to study its pulling or pushing effect in two specific directions. For example, in Fig. 2–8*a*, $\mathbf{F}$ is to be resolved into two components along the two members, defined by the u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of $\mathbf{F}$, one line parallel to u, and the other line parallel to v. These lines then intersect with the v and u axes, forming a parallelogram. The force components $\mathbf{F}_u$ and $\mathbf{F}_v$ are then established by simply joining the tail of $\mathbf{F}$ to the intersection points on the u and v axes, Fig. 2–8*b*. This parallelogram can then be reduced to a triangle, which represents the triangle rule, Fig. 2–8*c*. From this, the law of sines can then be applied to determine the unknown magnitudes of the components.

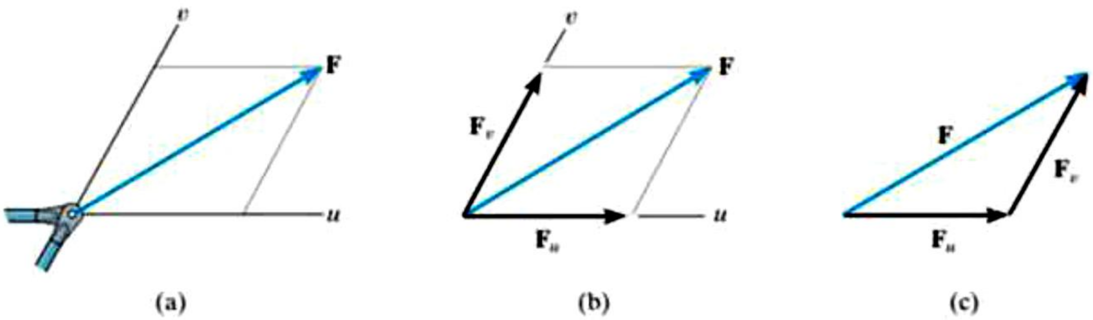


Fig. 2–8

Addition of Several Forces. If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force. For example, if three forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ act at a point O, Fig. 2–9, the resultant of any two of the forces is found, say, $\mathbf{F}_1 + \mathbf{F}_2$—and then this resultant is added to the third force, yielding the resultant of all three forces; i.e., $\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$. Using the parallelogram law to add more than two forces, as shown here, often requires extensive geometric and trigonometric calculation to determine the numerical values for the magnitude and direction of the resultant. Instead, problems of this type are easily solved by using the "rectangular-component method," which is explained in Sec. 2.4.

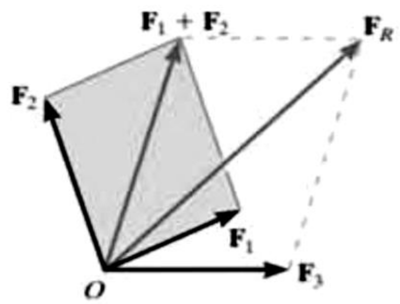
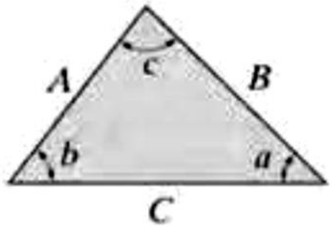
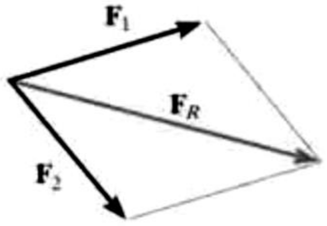


Fig. 2–9





(a)

Cosine law:

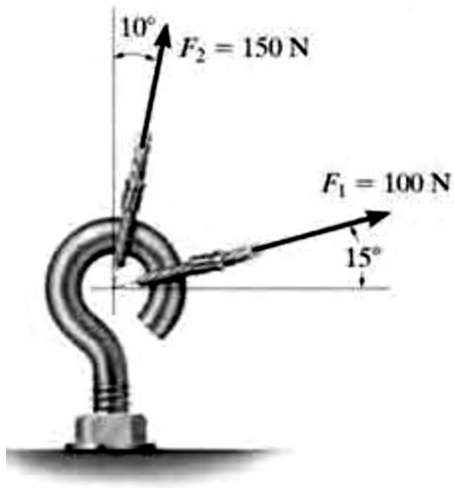
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

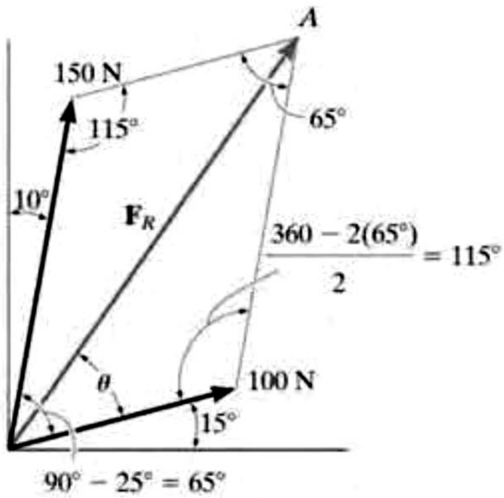
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Example //

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a)



(b)

Solution :

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point A, Fig. 2–11b. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

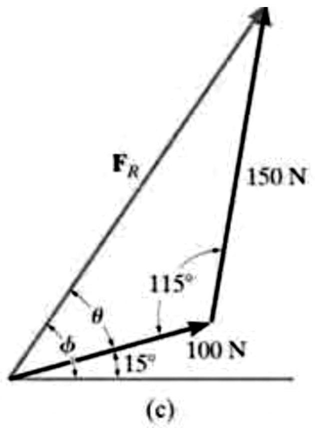


Fig. 2–11

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11c. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$
$$\theta = 39.8^\circ$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ \qquad \textit{Ans.}$$

<u>Example //</u>

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.

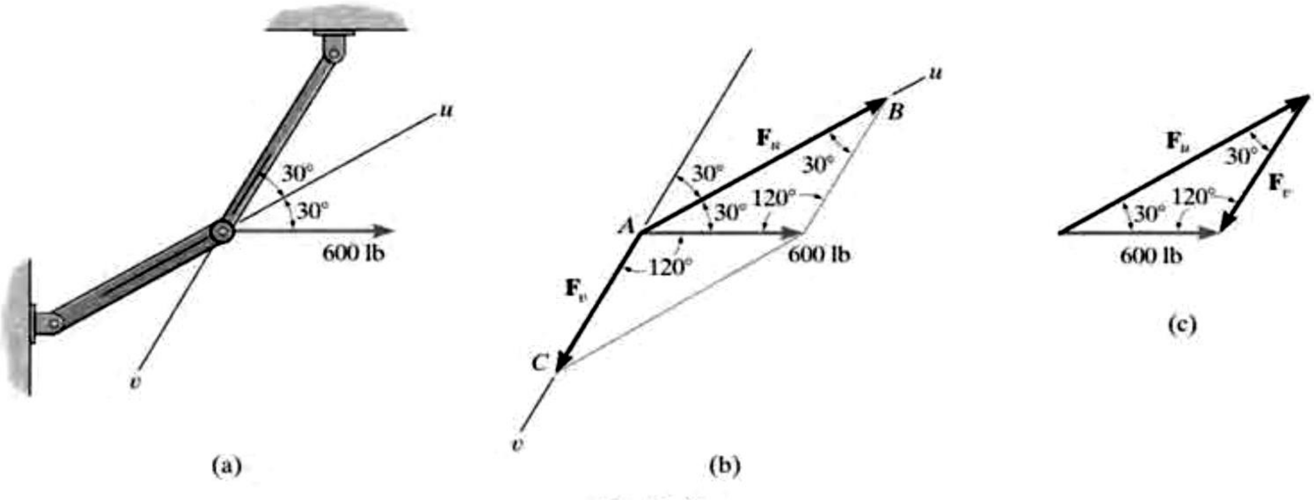


Fig. 2–12

<u>Solution :</u>

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the v axis until it intersects the u axis at point B, Fig. 2–12b. The arrow from A to B represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the u axis intersects the v axis at point C, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12c. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$

Example //

Determine the magnitude of the component force **F** in Fig. 2–13a and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive y axis.

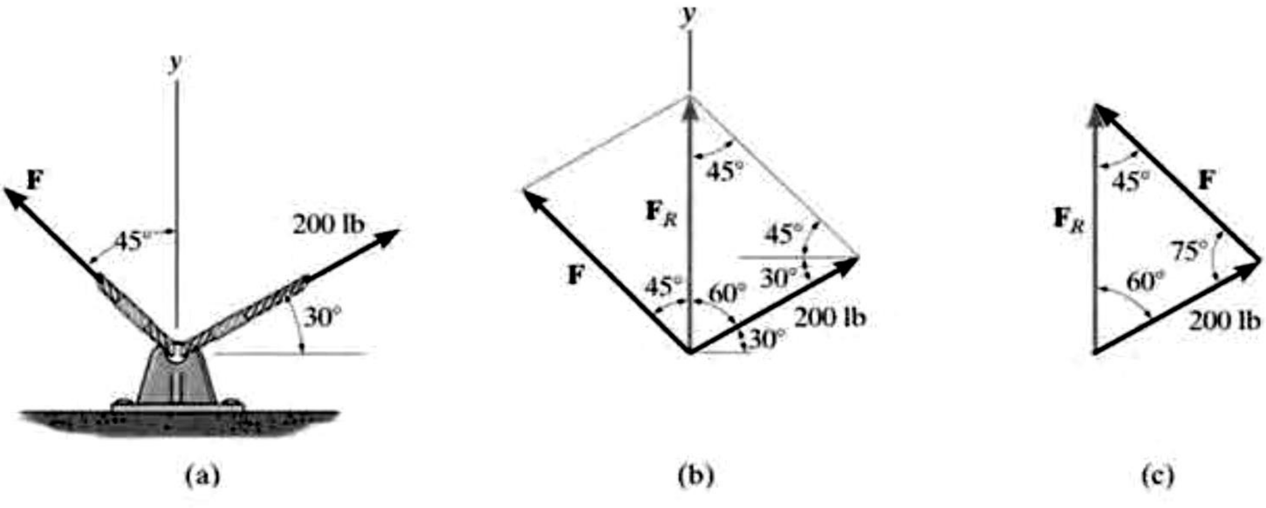


Fig. 2–13

Solution :

The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of $\mathbf{F}_R$ and $\mathbf{F}$ are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200\text{ lb}}{\sin 45^\circ}$$

$$F = 245\text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200\text{ lb}}{\sin 45^\circ}$$

$$F_R = 273\text{ lb} \qquad \textit{Ans.}$$