Lecture 5

Force System Resultants

Moment of a Force Scalar Formulation :

When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called a *torque*, but most often it is called the moment of a force or simply the *moment*.

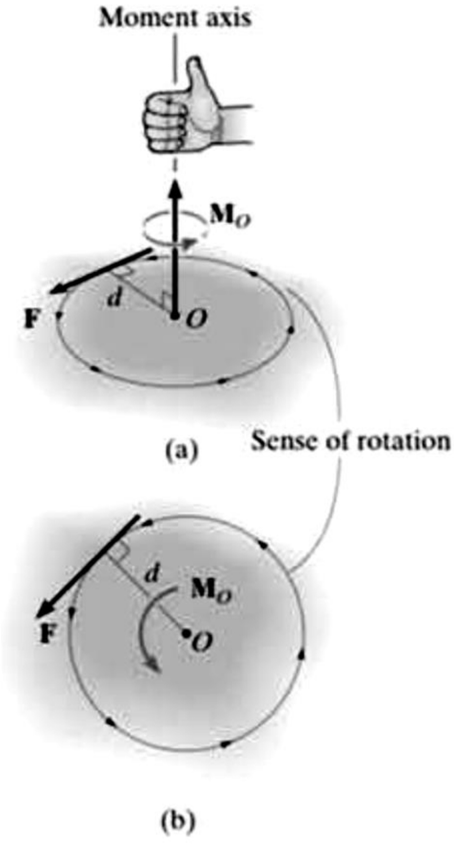


Fig. 4–2

Magnitude. The magnitude of $\mathbf{M}_O$ is

$$\boxed{M_O = Fd}$$

where d is the *moment arm* or *perpendicular distance* from the axis at point O to the line of action of the force. Units of moment magnitude consist of force times distance, e.g., N · m or lb · ft.

Direction. The direction of $\mathbf{M}_O$ is defined by its *moment axis*, which is perpendicular to the plane that contains the force **F** and its moment arm d. The right-hand rule is used to establish the sense of direction of $\mathbf{M}_O$. According to this rule, the natural curl of the fingers of the right hand, as they are drawn towards the palm, represent the tendency for rotation caused by the moment. As this action is performed, the thumb of the right hand will give the directional sense of $\mathbf{M}_O$, Fig. 4–2*a*. Notice that the moment vector is represented three-dimensionally by a curl around an arrow. In two dimensions this vector is represented only by the curl as in Fig. 4–2*b*. Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page.

Resultant Moment. For two-dimensional problems, where all the forces lie within the x–y plane, Fig. 4–3, the resultant moment $(\mathbf{M}_R)_o$ about point O (the z axis) can be determined by *finding the algebraic sum* of the moments caused by all the forces in the system. As a convention, we will generally consider *positive moments* as *counterclockwise* since they are directed along the positive z axis (out of the page). *Clockwise moments* will be *negative*. Doing this, the directional sense of each moment can be represented by a *plus or minus* sign. Using this sign convention, the resultant moment in Fig. 4–3 is therefore

$$\circlearrowleft + (M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$

If the numerical result of this sum is a positive scalar, $(\mathbf{M}_R)_O$ will be a counterclockwise moment, and if the result is negative, $(\mathbf{M}_R)_O$ will be a clockwise moment.

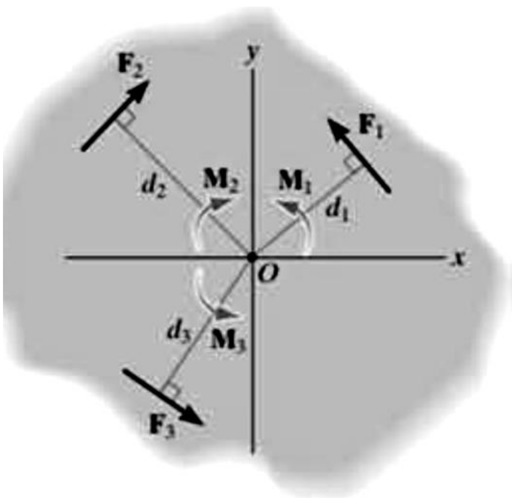


Fig. 4–3

Examples :

Determine the resultant moment of the four forces acting on the rod shown in Fig. 4–5 about point O.

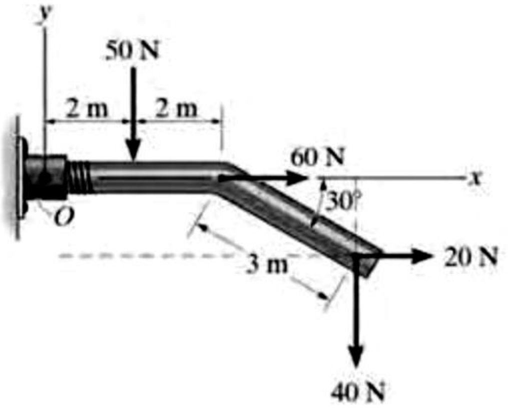


Fig. 4–5

Solution :

$$\circlearrowleft + M_{R_O} = \Sigma Fd;$$

$$M_{R_O} = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$
$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$

$$M_{R_O} = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\circlearrowright \qquad \textit{Ans.}$$

Principle of Moments :

It states that the moment of a force about a point is equal to the sum of the moments *of the components of the force about the point.* This theorem can be proven easily using the vector cross product since the cross product obeys the *distributive law*. For example, consider the moments of the force **F** and two of its components about point O. Fig. 4–16. Since $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

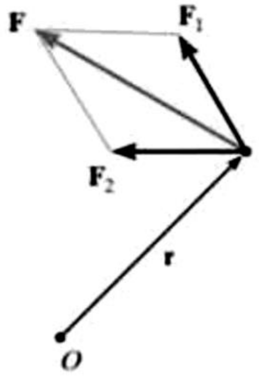
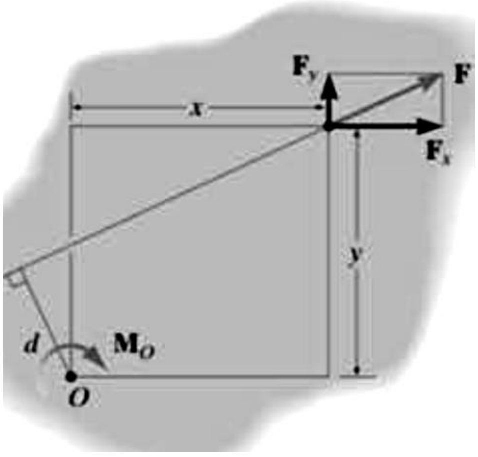


Fig 4–16

For two-dimensional problems, Fig. 4–17, we can use the principle of moments by resolving the force into its rectangular components and then determine the moment using a scalar analysis. Thus,

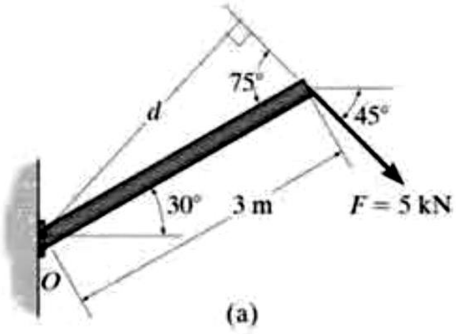
$$M_O = F_x y - F_y x$$

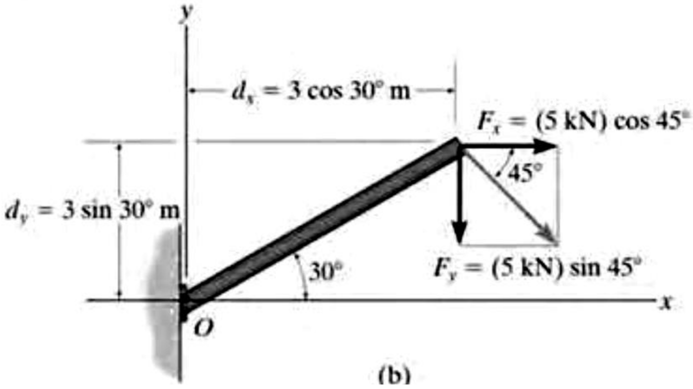


This method is generally easier than finding the same moment using $M_O = Fd$.

Example :

Determine the moment of the force as shown in figure about point O .





Solution I :

$$d = (3\text{ m})\sin 75° = 2.898\text{ m}$$

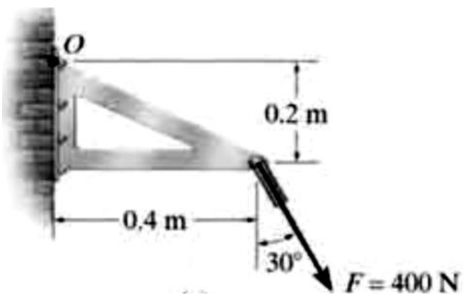
$$M_O = Fd = (5\text{kN})(2.898\text{ m}) = 14.5\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

Solution II :

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$
$$= -(5\cos 45°\text{ kN})(3\sin 30°\text{ m}) - (5\sin 45°\text{ kN})(3\cos 30°\text{ m})$$
$$= -14.5\text{ kN}\cdot\text{m} = 14.5\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

Example :

Force **F** acts at the end of the angle bracket shown in Fig. Determine the moment of the force about point O.



Solution :

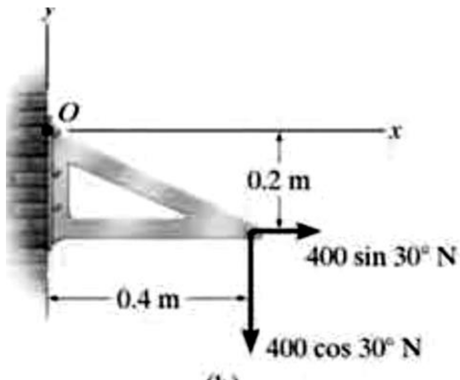
The force is resolved into its x and y components as shown in Fig. 4–19b, then

$$\circlearrowleft +M_O = 400 \sin 30^\circ \text{ N}(0.2 \text{ m}) - 400 \cos 30^\circ \text{ N}(0.4 \text{ m})$$

$$= -98.6 \text{ N}\cdot\text{m} = 98.6 \text{ N}\cdot\text{m} \circlearrowright$$

or

$$\mathbf{M}_O = \{-98.6\mathbf{k}\} \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$



(b)

Moment of a Couple :

A *couple* is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d, Fig. 4–25. Since the resultant force is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction.

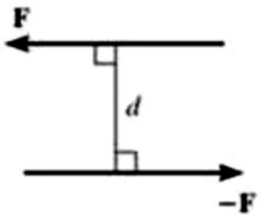


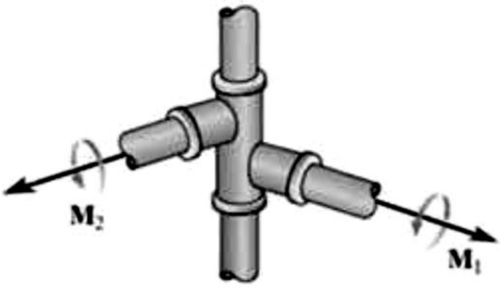
Fig. 4–25

The moment produced by a couple is called a *couple moment*. We can determine its value by finding the sum of the moments of both couple forces about *any* arbitrary point.

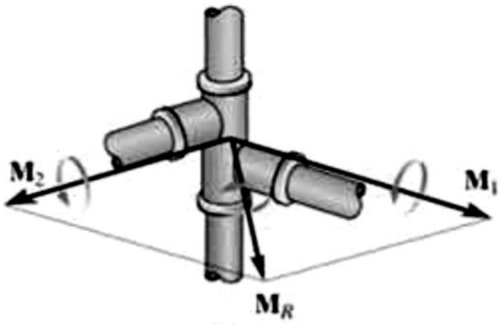
Resultant Couple Moment. Since couple moments are vectors, their resultant can be determined by vector addition. For example, consider the couple moments $\mathbf{M}_1$ and $\mathbf{M}_2$ acting on the pipe in Fig. 4–29*a*. Since each couple moment is a free vector, we can join their tails at any arbitrary point and find the resultant couple moment, $\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$ as shown in Fig. 4–29*b*.

If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$



(a)



(b)

Fig. 4–29

<u>Example :</u>

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

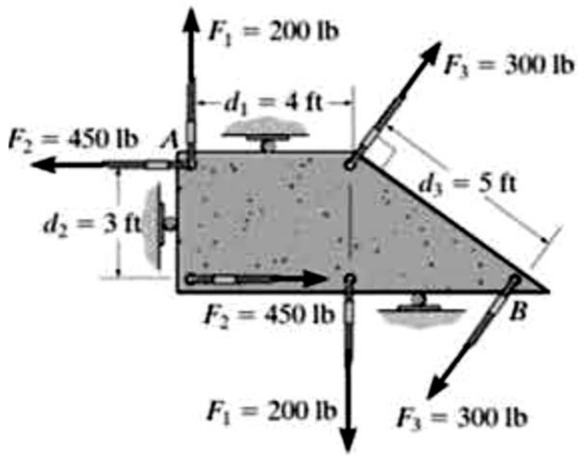


Fig. 4–30

Solution :

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\circlearrowleft +M_R = \Sigma M;\ M_R = -F_1d_1 + F_2d_2 - F_3d_3$$

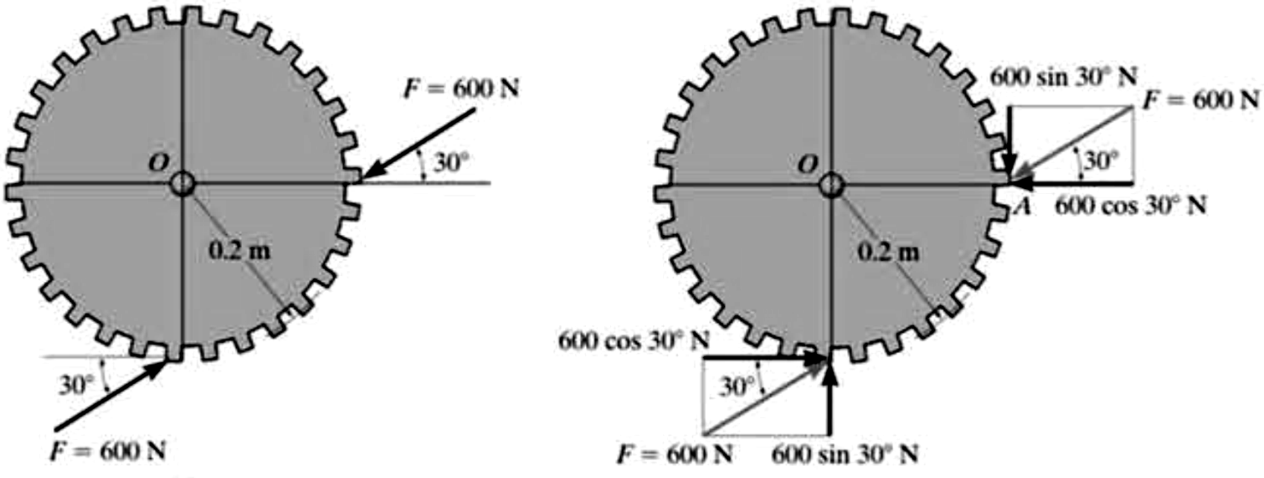
$$= (-200\text{ lb})(4\text{ ft}) + (450\text{ lb})(3\text{ ft}) - (300\text{ lb})(5\text{ ft})$$

$$= -950\text{ lb}\cdot\text{ft} = 950\text{ lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

Example :

Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.



(a) (b)

Solution :

$$\circlearrowleft + M = \Sigma M_O;\ M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$
$$= 43.9 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

or

$$\circlearrowleft + M = \Sigma M_A;\ M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$
$$= 43.9 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$