Lecture 4

Free-body diagram (FBD) :

Springs. If a *linearly elastic spring* (or cord) of undeformed length l_o is used to support a particle, the length of the spring will change in direct proportion to the force **F** acting on it, Fig. 3–1. A characteristic that defines the "elasticity" of a spring is the *spring constant* or *stiffness* k.

The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed) a distance $s = l - l_o$, measured from its *unloaded* position, is

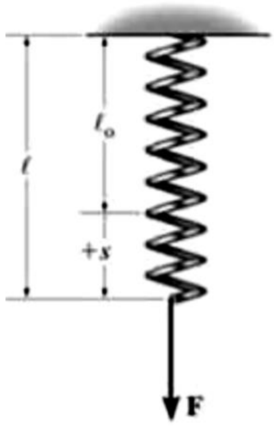


Fig. 3–1

$$\boxed{F = ks}$$

To apply the equation of equilibrium, we must account for *all* the known and unknown forces ($\Sigma\mathbf{F}$) which act *on* the particle. The best way to do this is to think of the particle as isolated and "free" from its surroundings. A drawing that shows the particle with *all* the forces that act on it is called a *free-body diagram (FBD)*.

$$\boxed{\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\end{aligned}}$$

These two equations can be solved for at most two unknowns, generally represented as angles and magnitudes of forces shown on the particle's free-body diagram.

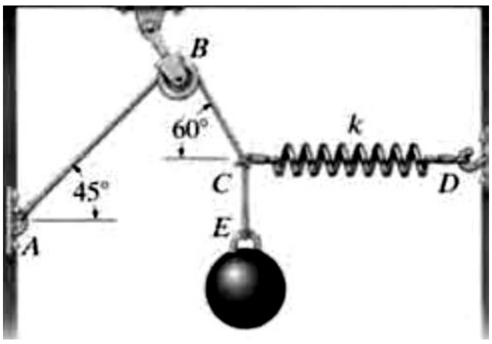
For example, consider the free-body diagram of the particle subjected to the two forces shown in Fig. 3–5. Here it is *assumed* that the *unknown force* **F** acts to the right to maintain equilibrium. Applying the equation of equilibrium along the x axis, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad +F + 10\text{ N} = 0$$

Fig. 3–5. Notice that if the $+x$ axis in Fig. 3–5 were directed to the left, both terms in the above equation would be negative, but again, after solving, $F = -10$ N, indicating that **F** would be directed to the left.

Example //

The sphere in Fig. 3–3*a* has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord CE, and the knot at C.



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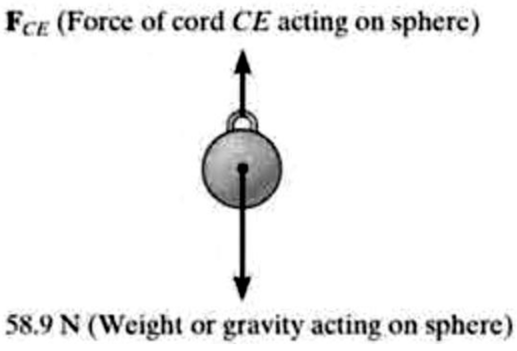
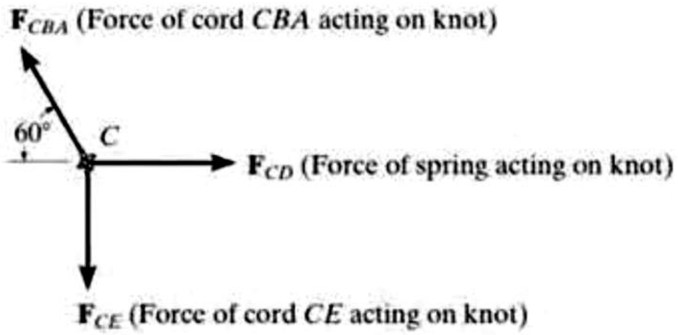
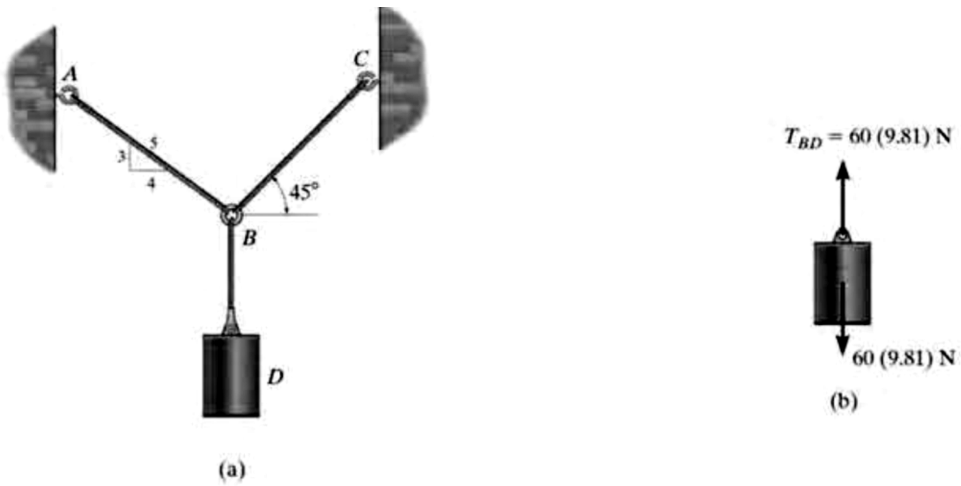




Examples:

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3-6a.



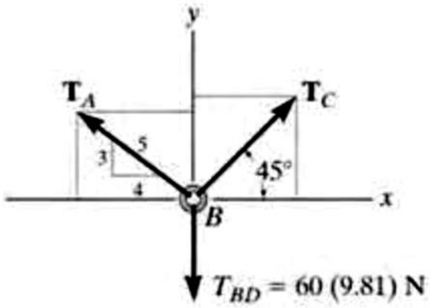
Solution//

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable BD to be $T_{BD} = 60(9.81)$ N, Fig. 3-6b. The forces in cables BA and BC can be determined by investigating the equilibrium of ring B. Its free-body diagram is shown in Fig. 3-6c. The magnitudes of $\mathbf{T}_A$ and $\mathbf{T}_C$ are unknown, but their directions are known.

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_C \cos 45^\circ - \left(\tfrac{4}{5}\right)T_A = 0$$

$$+\uparrow \Sigma F_y = 0; \quad T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0$$



Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

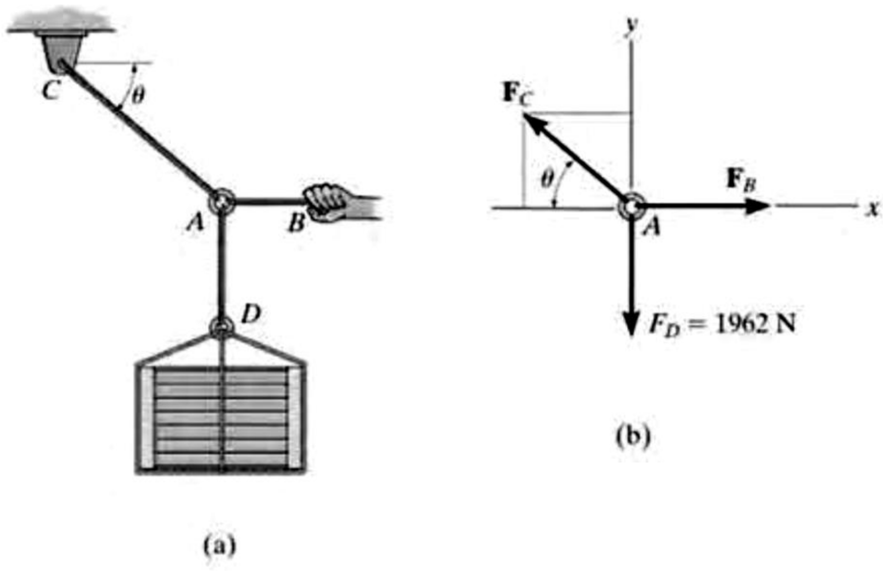
So that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

<u>**Example//**</u>



Solution//

Free-Body Diagram. We will study the equilibrium of ring A. There are three forces acting on it, Fig. 3-7*b*. The magnitude of $\mathbf{F}_D$ is equal to the weight of the crate, i.e., $F_D = 200\,(9.81)\text{ N} = 1962\text{ N} < 10\text{ kN}$.

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_C \cos\theta + F_B = 0; \quad F_C = \frac{F_B}{\cos\theta} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_C \sin\theta - 1962\text{ N} = 0 \qquad (2)$$

From Eq. (1), F_C is always greater than F_B since $\cos\theta \leq 1$. Therefore, rope AC will reach the maximum tensile force of 10 kN *before* rope AB. Substituting $F_C = 10$ kN into Eq. (2), we get

$$[10(10^3)\text{ N}]\sin\theta - 1962\text{ N} = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ \qquad \textit{Ans.}$$

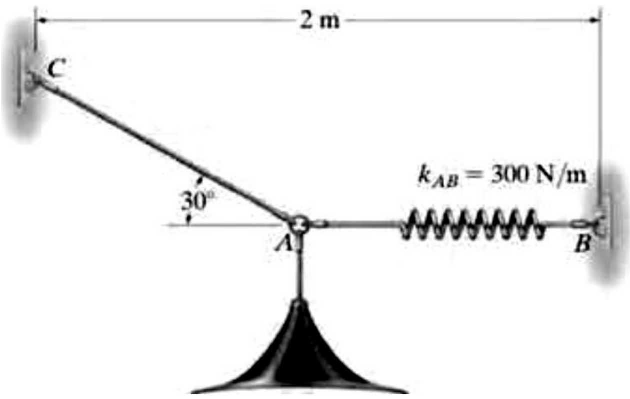
The force developed in rope AB can be obtained by substituting the values for θ and F_C into Eq. (1).

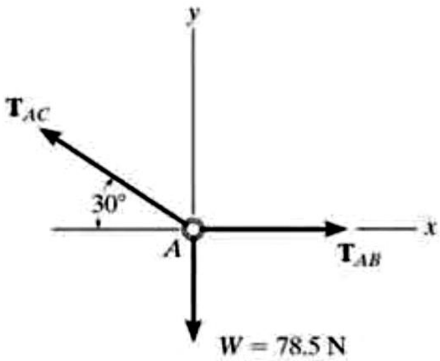
$$10(10^3)\text{ N} = \frac{F_B}{\cos 11.31^\circ}$$

$$F_B = 9.81\text{ kN}$$

Example//

Determine the required length of cord AC in Fig. 3–8*a* so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring AB is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.





SOLUTION

If the force in spring AB is known, the stretch of the spring can be found using $F = ks$. From the problem geometry, it is then possible to calculate the required length of AC.

Free-Body Diagram. The lamp has a weight $W = 8(9.81) = 78.5$ N and so the free-body diagram of the ring at A is shown in Fig. 3–8b.

Equations of Equilibrium. Using the x, y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{AB} - T_{AC} \cos 30^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{AC} \sin 30^\circ - 78.5 \text{ N} = 0$$

Solving, we obtain

$$T_{AC} = 157.0 \text{ N}$$

$$T_{AB} = 135.9 \text{ N}$$

The stretch of spring AB is therefore

$$T_{AB} = k_{AB} s_{AB}; \qquad 135.9 \text{ N} = 300 \text{ N/m}(s_{AB})$$

$$s_{AB} = 0.453 \text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$

$$l_{AB} = 0.4 \text{ m} + 0.453 \text{ m} = 0.853 \text{ m}$$

The horizontal distance from C to B, Fig. 3–8a, requires

$$2 \text{ m} = l_{AC} \cos 30^\circ + 0.853 \text{ m}$$

$$l_{AC} = 1.32 \text{ m} \qquad \textit{Ans.}$$