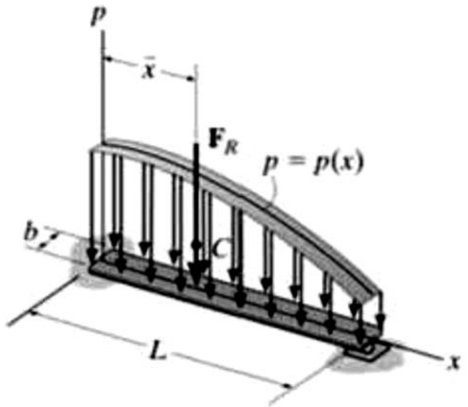
Lecture 6

Reduction of a Simple Distributed Loading :

Sometimes, a body may be subjected to a loading that is distributed over its surface. For example, the pressure of the wind on the face of a sign, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all *distributed loadings*. The pressure exerted at each point on the surface indicates the intensity of the loading. It is measured using pascals Pa (or N/m^2) in SI units or lb/ft^2 in the U.S. Customary system.

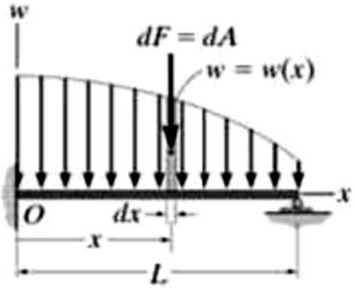
Uniform Loading Along a Single Axis. The most common type of distributed loading encountered in engineering practice is generally uniform along a single axis.*

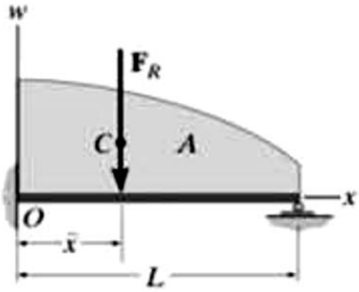


Magnitude of Resultant Force. :

the magnitude of $\mathbf{F}_R$ is equivalent to the sum of all the forces in the system. In this case integration must be used since there is an infinite number of parallel forces $d\mathbf{F}$ acting on the beam,

$$F_R = \int_L w(x)\,dx = \int_A dA = A$$





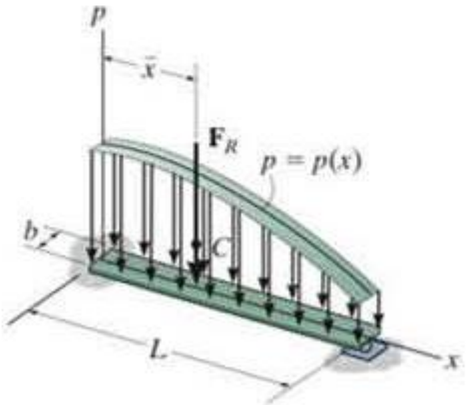
dF: is acting on an element of length dx

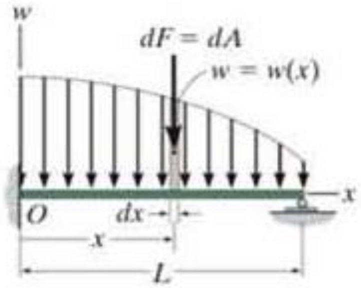
w(x) : is a force per unit length .
dF = w(x) dx = dA

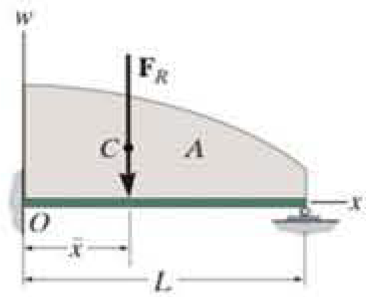
Location of Resultant Force.

the location $\bar{x}$ of the line of action of $\mathbf{F}_R$ can be determined by equating the moments of the force resultant and the parallel force distribution about point O (the y axis).

$$\bar{x} = \frac{\int_L x w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$



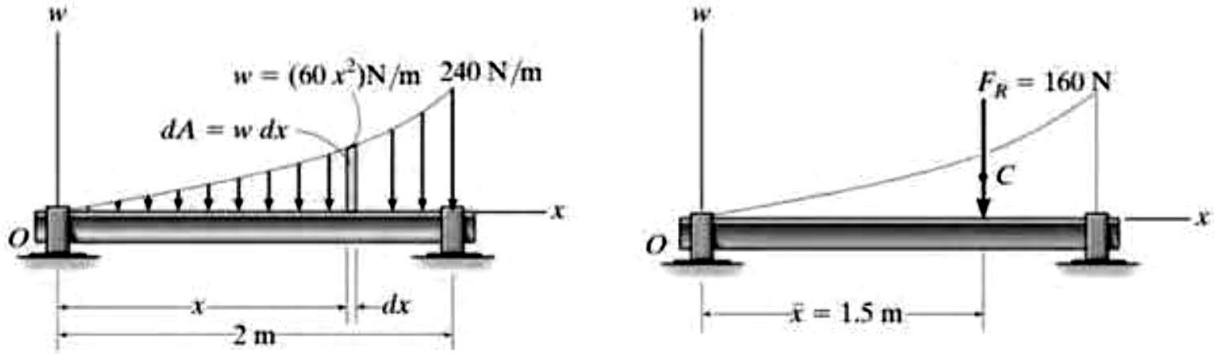




This coordinate $\bar{x}$, locates the geometric center or *centroid* of the *area* under the distributed loading.

<u>Example //</u>

Determine the magnitude and location of the equivalent resultant force acting on the shaft in Fig.



Solution :

The differential element has an area $dA = w\,dx = 60x^2\,dx$. Applying

$+\downarrow F_R = \Sigma F;$

$$F_R = \int_A dA = \int_0^{2\text{ m}} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Bigg|_0^{2\text{ m}} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

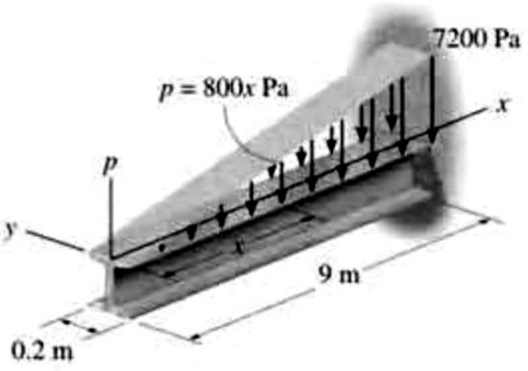
$$= 160\text{ N}$$

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^{2\text{ m}} x(60x^2)\,dx}{160\text{ N}} = \frac{60\left(\frac{x^4}{4}\right)\Big|_0^{2\text{ m}}}{160\text{ N}} = \frac{60\left(\frac{2^4}{4} - \frac{0^4}{4}\right)}{160\text{ N}}$$

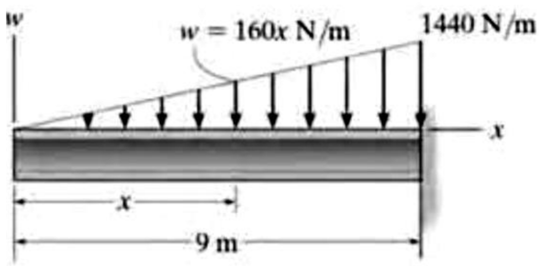
$$= 1.5\text{ m}$$

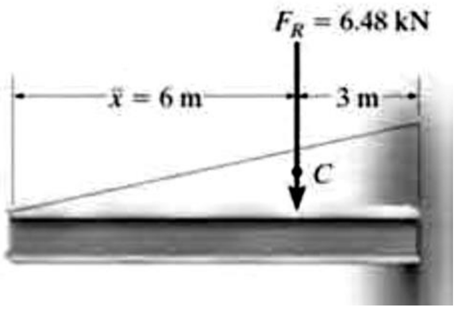
Example //

A distributed loading of $p = (800x)$ Pa acts over the top surface of the beam shown in Fig. 4–50a. Determine the magnitude and location of the equivalent resultant force.



Solution //





$$w = (800x\ \mathrm{N/m^2})(0.2\ \mathrm{m})$$
$$= (160x)\ \mathrm{N/m}$$

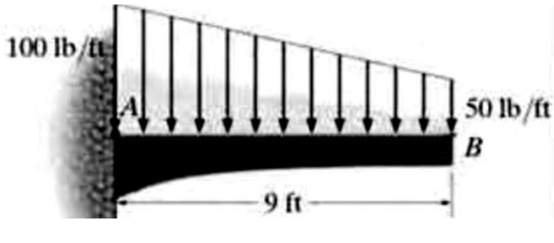
At $x = 9$ m, note that $w = 1440$ N/m.

$F_R = \frac{1}{2}(9\ \mathrm{m})(1440\ \mathrm{N/m}) = 6480\ \mathrm{N} = 6.48\ \mathrm{kN}$

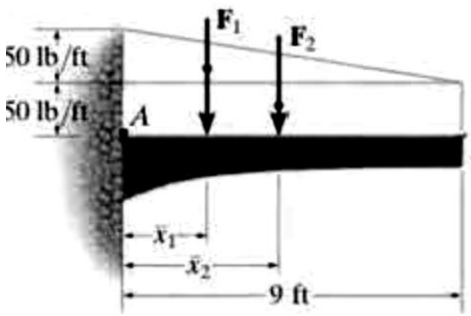
$\bar{x} = 9\ \mathrm{m} - \frac{1}{3}(9\ \mathrm{m}) = 6\ \mathrm{m}$

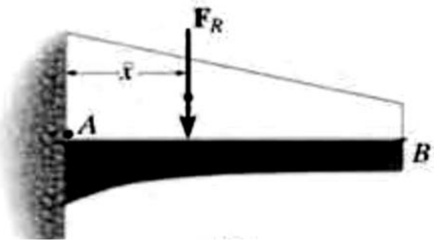
Example//

The granular material exerts the distributed loading on the beam as shown in Fig. 4–51*a*. Determine the magnitude and location of the equivalent resultant of this load.



Solution //





$$F_1 = \tfrac{1}{2}(9\text{ ft})(50\text{ lb/ft}) = 225\text{ lb}$$

$$F_2 = (9\text{ ft})(50\text{ lb/ft}) = 450\text{ lb}$$

$$\bar{x}_1 = \tfrac{1}{3}(9\text{ ft}) = 3\text{ ft}$$

$$\bar{x}_2 = \tfrac{1}{2}(9\text{ ft}) = 4.5\text{ ft}$$

$$+\downarrow F_R = \Sigma F; \qquad F_R = 225 + 450 = 675\text{ lb}$$

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad \bar{x}(675) = 3(225) + 4.5(450)$$

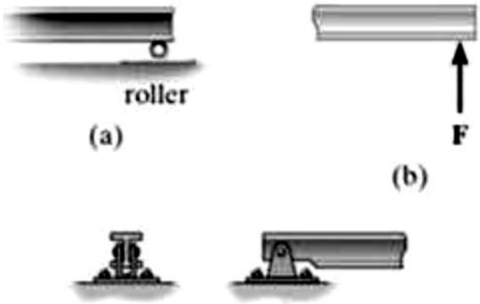
$$\bar{x} = 4\text{ ft}$$

Equilibrium of a Rigid Body :

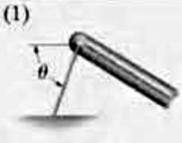
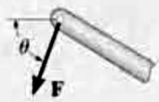
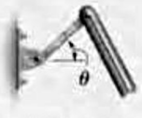
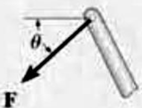
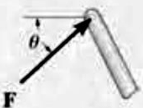
Free-Body Diagrams

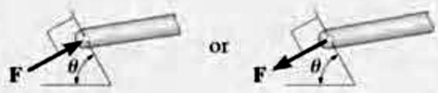
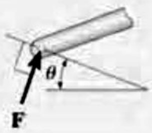
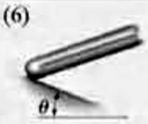
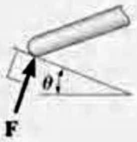
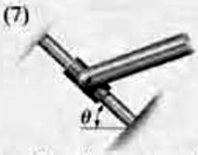
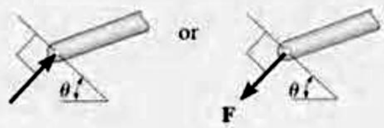
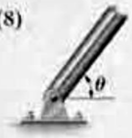
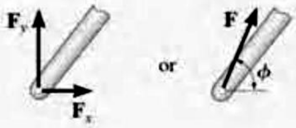
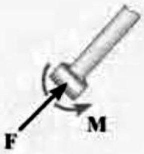
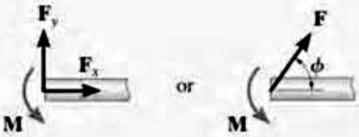
Support Reactions. Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule,

- If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.
- If rotation is prevented, a couple moment is exerted on the body.



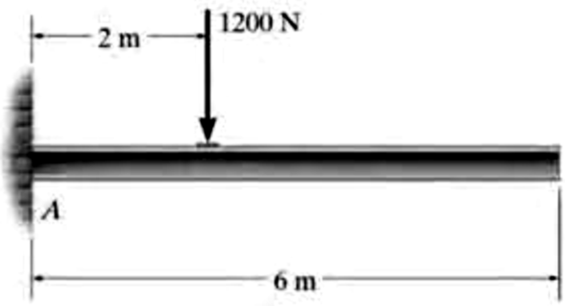
(a) (b) (c)

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

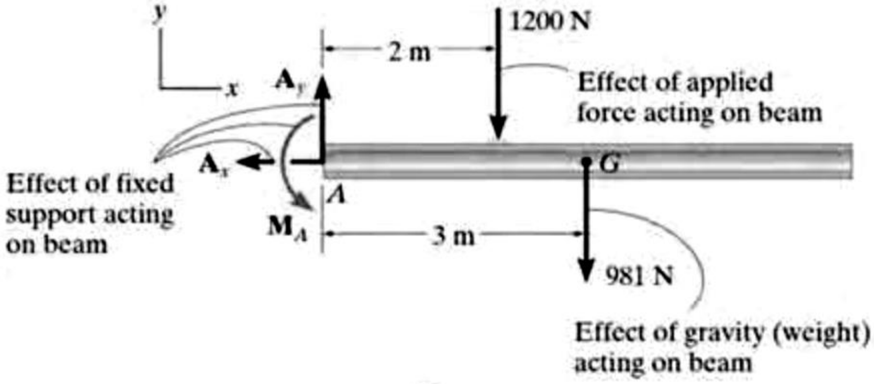
Types of Connection	Reaction	Number of Unknowns
(4)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

<u>Example//</u>

Draw the free-body diagram of the uniform beam shown in Fig. The beam has a mass of 100 kg.

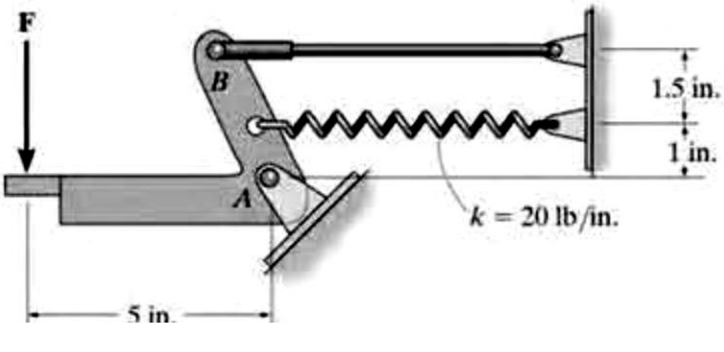


<u>Solution //</u>

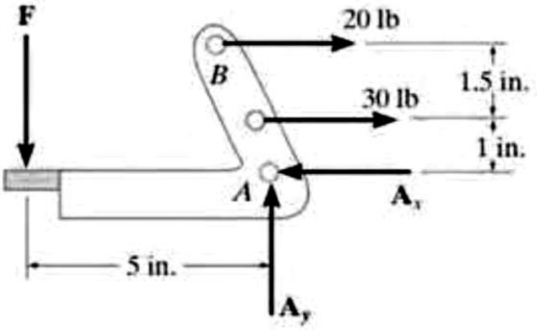


<u>Example//</u>

Draw the free-body diagram of the foot lever shown in Fig. 5–8*a*. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at *B* is 20 lb.

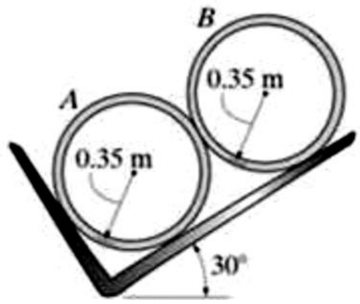


<u>Solution //</u>

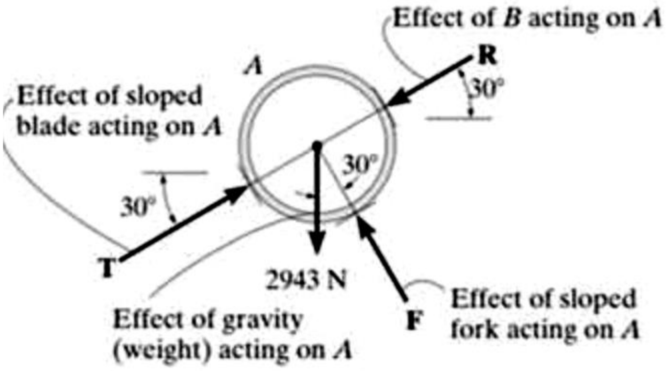


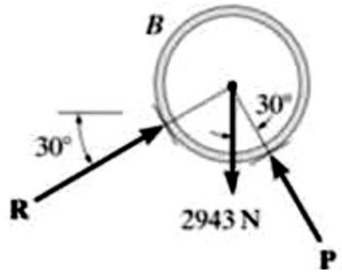
Example//

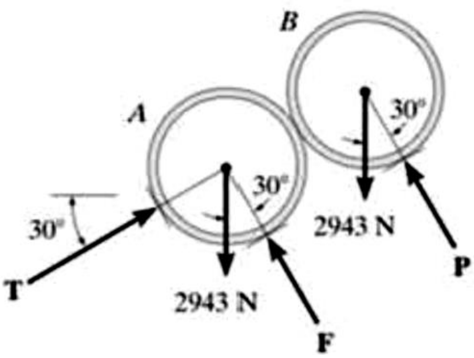
Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. 5–9*a*. Draw the free-body diagrams for each pipe and both pipes together.



Solution //

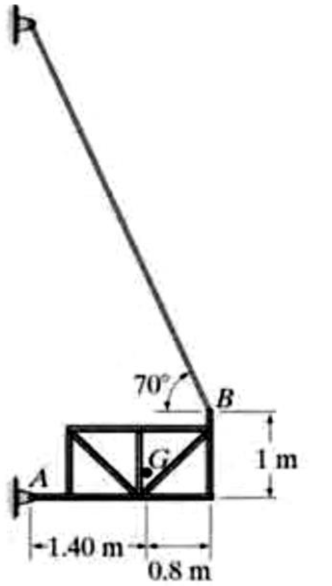




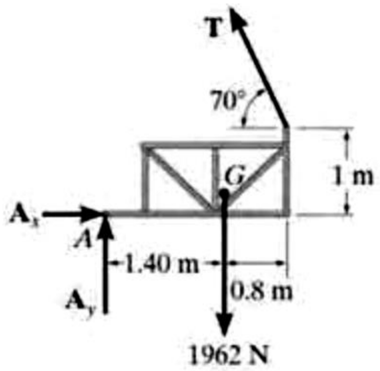


Example//

Draw the free-body diagram of the unloaded platform that is suspended off the edge of the oil rig shown in Fig. 5–10*a*. The platform has a mass of 200 kg.



Solution //



Equations of Equilibrium :

When the body is subjected to a system of forces, which all lie in the x-y plane, then the forces can be resolved into their x and y components. Consequently, the conditions for equilibrium in two dimensions are :

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

Here ΣF_x and ΣF_y represent, respectively, the algebraic sums of the x and y components of all the forces acting on the body, and ΣM_O represents the algebraic sum of the couple moments and the moments of all the force components about the z axis, which is perpendicular to the x–y plane and passes through the arbitrary point O.

Coplanar force equilibrium problems for a rigid body can be solved using the following procedure.

Free-Body Diagram.

- Establish the x, y coordinate axes in any suitable orientation.
- Draw an outlined shape of the body.
- Show all the forces and couple moments acting on the body.
- Label all the loadings and specify their directions relative to the x or y axis. The sense of a force or couple moment having an *unknown* magnitude but known line of action can be *assumed*.
- Indicate the dimensions of the body necessary for computing the moments of forces.

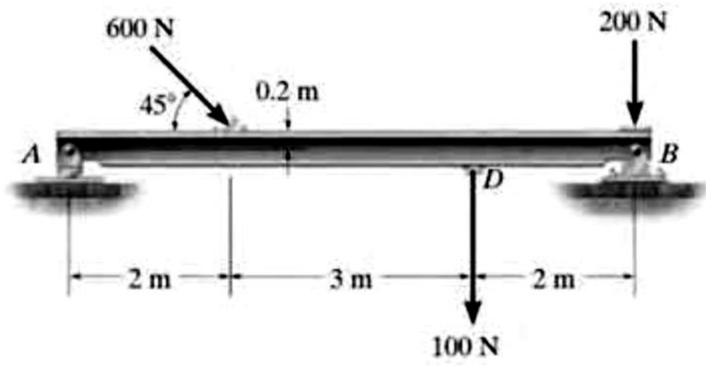
Equations of Equilibrium.

- Apply the moment equation of equilibrium, $\Sigma M_O = 0$, about a point (O) that lies at the intersection of the lines of action of two unknown forces. In this way, the moments of these unknowns are zero about O, and a *direct solution* for the third unknown can be determined.

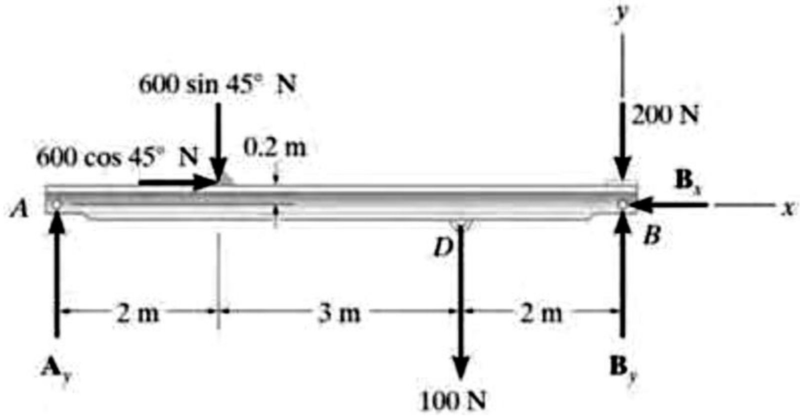
- When applying the force equilibrium equations, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest resolution of the forces into their x and y components.
- If the solution of the equilibrium equations yields a negative scalar for a force or couple moment magnitude, this indicates that the sense is opposite to that which was assumed on the free-body diagram.

Example //

Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. 5–12a. Neglect the weight of the beam.



Solution //



Equations of Equilibrium. Summing forces in the x direction yields

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m}) - (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

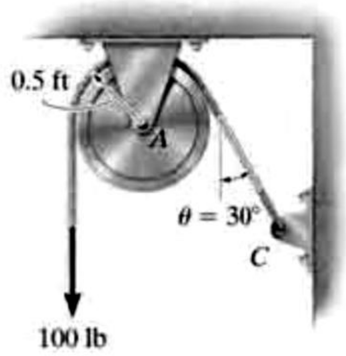
$$A_y = 319 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

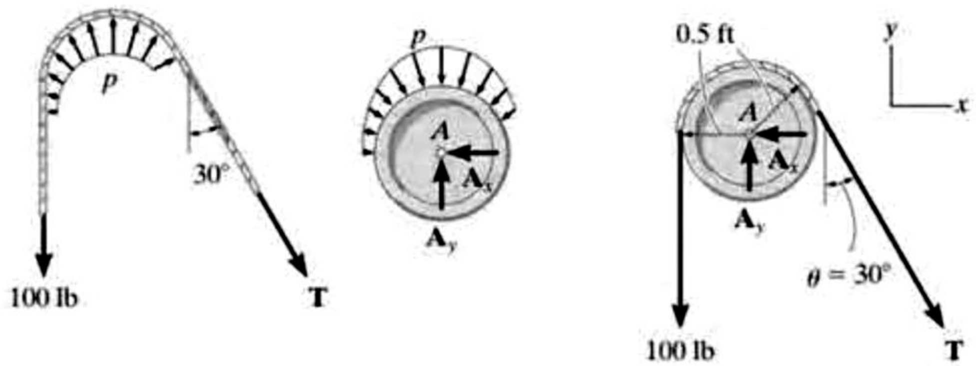
$$B_y = 405 \text{ N}$$

Example //

The cord shown in Fig. 5–13*a* supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



Solution //



$\circlearrowleft + \Sigma M_A = 0;$ $\quad 100 \text{ lb } (0.5 \text{ ft}) - T(0.5 \text{ ft}) = 0$

$$T = 100 \text{ lb}$$

Using the result,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -A_x + 100 \sin 30° \text{ lb} = 0$

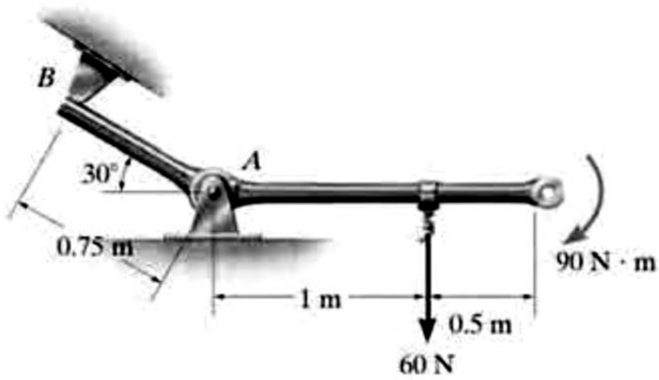
$$A_x = 50.0 \text{ lb}$$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 100 \text{ lb} - 100 \cos 30° \text{ lb} = 0$

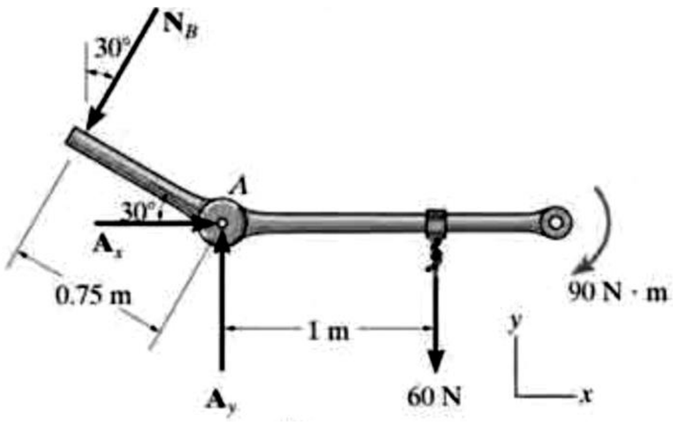
$$A_y = 187 \text{ lb}$$

Example //

The member shown in Fig. 5–14*a* is pin-connected at *A* and rests against a smooth support at *B*. Determine the horizontal and vertical components of reaction at the pin *A*.



Solution //



$$\circlearrowleft + \Sigma M_A = 0; \quad -90\ \mathrm{N\cdot m} - 60\ \mathrm{N}(1\ \mathrm{m}) + N_B(0.75\ \mathrm{m}) = 0$$

$$N_B = 200\ \mathrm{N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 200 \sin 30^\circ\ \mathrm{N} = 0$$

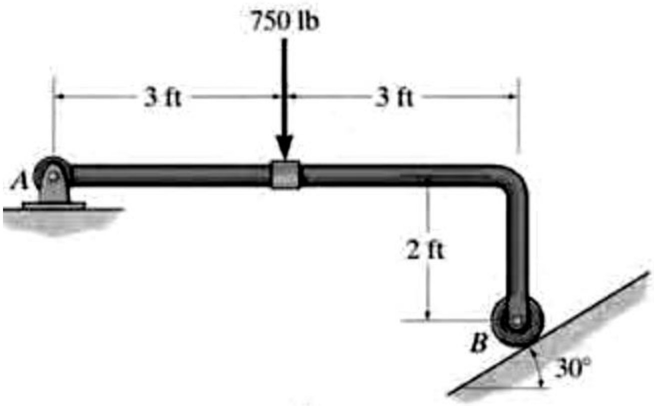
$$A_x = 100\ \mathrm{N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200 \cos 30^\circ\ \mathrm{N} - 60\ \mathrm{N} = 0$$

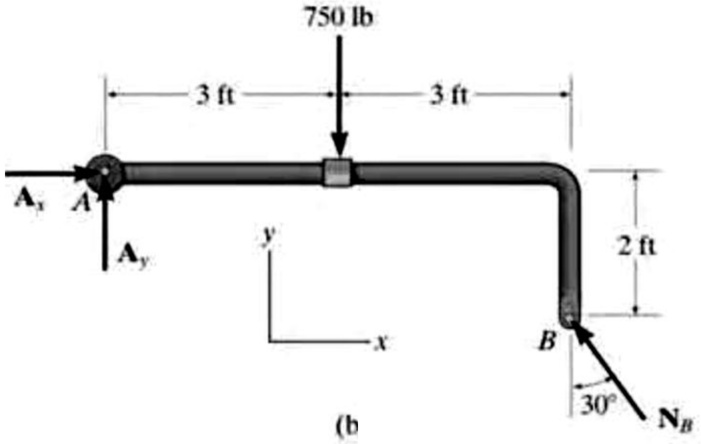
$$A_y = 233\ \mathrm{N}$$

Example //

Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at the roller B in Fig. 5–16a.



Solution //



$\circlearrowleft + \Sigma M_A = 0;$

$$[N_B \cos 30°](6 \text{ ft}) - [N_B \sin 30°](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0$$

$$N_B = 536.2 \text{ lb} = 536 \text{ lb}$$

Using this result,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - (536.2 \text{ lb}) \sin 30° = 0$

$A_x = 268 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + (536.2 \text{ lb}) \cos 30° - 750 \text{ lb} = 0$

$A_y = 286 \text{ lb}$