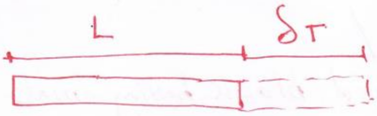
Thermal strain :-



$$\boxed{\varepsilon_T = \alpha \, \Delta T}$$

where

ε_T = thermal strain / ε_T → + expansion تمدد ; → − contraction تقلص

α = ~~Modulus of~~ coefficient of thermal expansion of the material is it made of body $\frac{1}{C^\circ}$, $\frac{1}{F^\circ}$ ~~فهو~~ فهرنهايت

ΔT = The change in temperature.

$$\Delta T = T_{final} - T_{initial}$$

ΔT → + increase ; → − decrease

* في حال تعرض الجسم لأحمال عليه تحميل بثلاث اتجاهات بالإضافة الى الحرارة بثلاثة اتجاهات فإن الانفعالات ستكون:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \mp \alpha \Delta T$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_z}{E} \pm \alpha \Delta T$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \pm \alpha \Delta T$$

Thermal deformation :- التشوه الحراري

$$\varepsilon_t = \alpha \Delta T$$

$$\frac{\delta_T}{L} = \alpha \Delta T$$

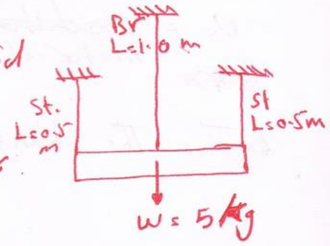
$$\delta_T = \alpha \Delta T L$$

δ_T = thermal deformation

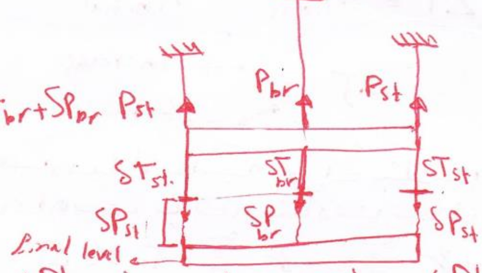
Example/

· A rigid block having a mass of 5 Kg is supported by three rods symmetrically placed, as shown in fig. determine the stress in each rod after a temperature rise of 40 °C. the lower ends of the rods are assumed to have been at the same level before the block was attached and the temperature changed. that symmetry dictates that the block will remain horizontal. Use the data in the accompanying table.

	Steel rod	Bronze rod
area $(mm)^2$	500	900
E (N/m^2)	200×10^9	83×10^9
α $\frac{1}{C^o}$	11.7×10^{-6}	18.9×10^{-6}



$$\delta T_{st} + \delta P_{st} = \delta T_{br} + \delta P_{br}$$



$$(\alpha L \Delta T)_{st} + \left(\frac{PL}{AE}\right)_{st} = (\alpha L \Delta T)_{br} + \left(\frac{PL}{AE}\right)_{br}$$

$$(11.7 \times 10^{-6}) \times (0.5)(40) + \frac{P_s \times (0.5)}{(500 \times 10^{-6})(200 \times 10^9)} = (18.9 \times 10^{-6})(1.0) \times (40)$$

$$+ \frac{P_{br} \times (1)}{(900 \times 10^{-6})(83 \times 10^9)}$$

$$5 \times 10^{-9} P_{st} - 1.34 \times 10^{-8} P_b = 5.22 \times 10^{-4}$$

$$\Sigma F_y = 0$$

$$2P_{st} + P_{br} = \underbrace{5 \times 1000 \times 9.81}_{49.05 \times 10^3 N} \Longrightarrow P_{br} = -24990 N$$

$$P_{st} = +37000 N$$

$P_s = 37.0 kN$

$P_b = -25.0 kN$

$$\sigma_{st} = \frac{P}{A}$$

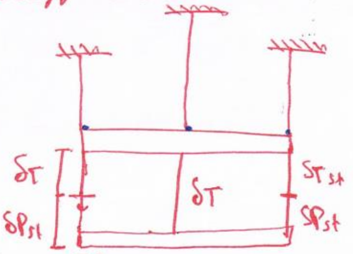
$$= \frac{37.0 \times 10^3}{500 \times 10^{-6}} \Longrightarrow \sigma_{st} = 74.0\ MN/m^2 \text{ (tension)}$$

$$\sigma_{br} = \frac{25.0 \times 10^3}{900 \times 10^{-6}} \Longrightarrow \sigma_{br} = 27.8\ MN/m^2 \text{ (compression)}$$

Example

Using the data in the previous example determine the temperature rise necessary to cause all the applied load to be supported by the steel rods.

Sol



$$\delta T_{br} = \delta T_{st} + \delta P_{st}$$

$$(\alpha \Delta T L)_{br} = (\alpha L \Delta T)_{st} + \left(\frac{PL}{AE}\right)_{st}$$

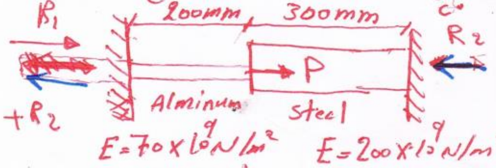
$$\frac{1}{2} \times 5000 \times 9.81$$

$$(18.9 \times 10^{-6})(1)(\Delta T) = (11.7 \times 10^{-6})(0.5)(\Delta T) + \frac{24.53 \times 10^3 (0.5)}{(500 \times 10^{-6})(200 \times 10^9)}$$

$$\Delta T = 9.4\ ^\circ C$$

Example

The composite bar shown in fig. is firmly attached to unyielding supports. An axial load $P = 200\ kN$ is applied at $20\ ^\circ C$. Find the stress in each material at $60\ ^\circ C$. Assume $\alpha = 11.7 \times 10^{-6}\ \frac{1}{C^\circ}$ for steel, $23 \times 10^{-6}\ \frac{1}{C^\circ}$ for Aluminum.



Sol

$$-R_1 + R_2 + 200 = 0$$

$$200 = R_1 + R_2$$

$$(\delta T)_A + (\delta T)_{st} = (\delta P)_{al.} + (\delta P)_{st}$$

$$(\alpha \Delta T L)_{al.} + (\alpha \Delta T L)_{st} = \left(\frac{PL}{AE}\right)_{al.} + \left(\frac{PL}{AE}\right)_{st.}$$

$$\boxed{P_{al} = R_1}$$

$$(23 \times 10^{-6} \times 40 \times 200 \times 10^{-3}) + (11.7 \times 10^{-6} \times 40 \times 300 \times 10^{-3})$$

$$= \frac{(R_1 + 200 \times 10^3) \times 300 \times 10^{-3}}{1200 \times 10^{-6} \times 200 \times 10^9} + \frac{R_1 \times 200 \times 10^{-3}}{10^{-6} \times 900 \times 70 \times 10^9}$$

$$~~P_{st} = R_1 + P~~$$

$R_1 = 16800\ N$

$R_1 = 16.8\ kN$

$$\sigma_{al} = \frac{R_1}{A} = \frac{16.8 \times 10^3}{900 \times 10^{-6}} = 18.6\ MPa$$

$$\sigma_{st} = \frac{R_1 + P}{A} = \frac{(16.8 + 200) \cdot 10^3}{1200} = 180.7\ MPa$$