Chapter 1

Non-Newtonian fluid behaviour

1.1 Introduction

One may classify fluids in two different ways; either according to their response to the externally applied pressure or according to the effects produced under the action of a shear stress. The first scheme of classification leads to the so called ‘compressible’ and ‘incompressible’ fluids, depending upon whether or not the volume of an element of fluid is dependent on its pressure. While compressibility influences the flow characteristics of gases, liquids can normally be regarded as incompressible and it is their response to shearing which is of greater importance. In this chapter, the flow characteristics of single phase liquids, solutions and pseudo-homogeneous mixtures (such as slurries, emulsions, gas–liquid dispersions) which may be treated as a continuum if they are stable in the absence of turbulent eddies are considered depending upon their response to externally imposed shearing action.

1.2 Classification of fluid behaviour

1.2.1 Definition of a Newtonian fluid

Consider a thin layer of a fluid contained between two parallel planes a distance dy apart, as shown in Figure 1.1. Now, if under steady state conditions, the fluid is subjected to a shear by the application of a force F as shown, this will be balanced by an equal and opposite internal frictional force in the fluid. For an incompressible Newtonian fluid in laminar flow, the resulting shear stress is equal to the product of the shear rate and the viscosity of the fluid medium. In this simple case, the shear rate may be expressed as the velocity gradient in the direction perpendicular to that of the shear force, i.e.

$$\frac{F}{A} = \tau_{yx} = \mu \left(-\frac{\mathrm{d}V_x}{\mathrm{d}y} \right. = \mu \dot{\gamma}_{yx} \tag{1.1}$$

Note that the first subscript on both τ and $\dot{\gamma}$ indicates the direction normal to that of shearing force, while the second subscript refers to the direction of the force and the flow. By considering the equilibrium of a fluid layer, it can

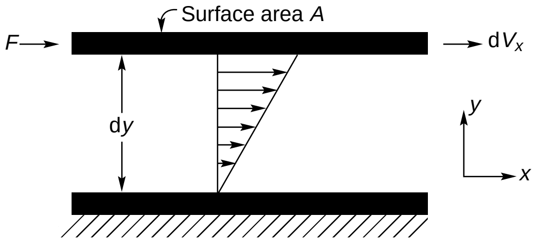

Figure 1.1 *Schematic representation of unidirectional shearing flow*

readily be seen that at any shear plane there are two equal and opposite shear stresses–a positive one on the slower moving fluid and a negative one on the faster moving fluid layer. The negative sign on the right hand side of equation (1.1) indicates that τ_{yx} is a measure of the resistance to motion. One can also view the situation from a different standpoint as: for an incompressible fluid of density ρ, equation (1.1) can be written as:

$$\tau_{yx} = -\frac{\mu}{\rho}\frac{\mathrm{d}}{\mathrm{d}y}(\rho V_x) \tag{1.2}$$

The quantity 'ρV_x' is the momentum in the x-direction per unit volume of the fluid and hence τ_{yx} represents the momentum flux in the y-direction and the negative sign indicates that the momentum transfer occurs in the direction of decreasing velocity which is also in line with the Fourier's law of heat transfer and Fick's law of diffusive mass transfer.

The constant of proportionality, μ (or the ratio of the shear stress to the rate of shear) which is called the Newtonian viscosity is, by definition, independent of shear rate ($\dot{\gamma}_{yx}$) or shear stress (τ_{yx}) and depends only on the material and its temperature and pressure. The plot of shear stress (τ_{yx}) against shear rate ($\dot{\gamma}_{yx}$) for a Newtonian fluid, the so-called 'flow curve' or 'rheogram', is therefore a straight line of slope, μ, and passing through the origin; the single constant, μ, thus completely characterises the flow behaviour of a Newtonian fluid at a fixed temperature and pressure. Gases, simple organic liquids, solutions of low molecular weight inorganic salts, molten metals and salts are all Newtonian fluids. The shear stress–shear rate data shown in Figure 1.2 demonstrate the Newtonian fluid behaviour of a cooking oil and a corn syrup; the values of the viscosity for some substances encountered in everyday life are given in Table 1.1.

Figure 1.1 and equation (1.1) represent the simplest case wherein the velocity vector which has only one component, in the x-direction varies only in the y-direction. Such a flow configuration is known as simple shear flow. For the more complex case of three dimensional flow, it is necessary to set up the appropriate partial differential equations. For instance, the more general case of an incompressible Newtonian fluid may be expressed – for the x-plane – as

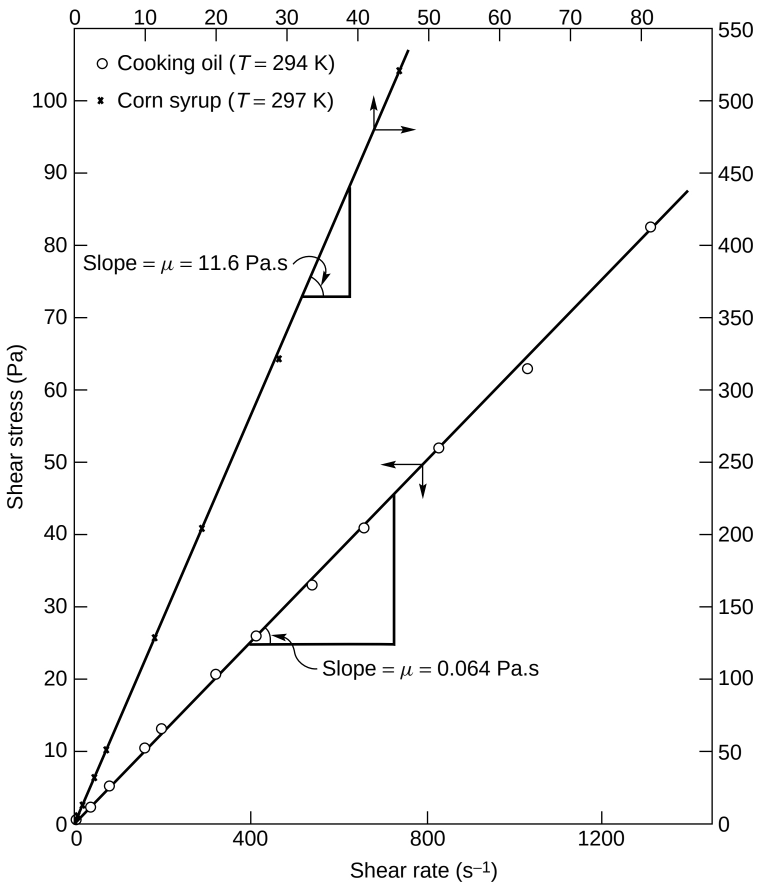

Figure 1.2 *Typical shear stress–shear rate data for a cooking oil and a corn syrup*

follows [Bird *et al.*, 1960, 1987]:

$$\tau_{xx} = -2\mu \frac{\partial V_x}{\partial x} + \frac{2}{3}\mu \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) \tag{1.3}$$

$$\tau_{xy} = -\mu \left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right) \tag{1.4}$$

$$\tau_{xz} = -\mu \left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right) \tag{1.5}$$

Similar sets of equations can be drawn up for the forces acting on the y- and z-planes; in each case, there are two (in-plane) shearing components and a

Table 1.1 *Typical viscosity values at room temperature*

Substance	μ (mPa·s)
Air	10^{-2}
Benzene	0.65
Water	1
Molten sodium chloride (1173 K)	1.01
Ethyl alcohol	1.20
Mercury (293 K)	1.55
Molten lead (673 K)	2.33
Ethylene glycol	20
Olive oil	100
Castor oil	600
100% Glycerine (293 K)	1500
Honey	10^4
Corn syrup	10^5
Bitumen	10^{11}
Molten glass	10^{15}

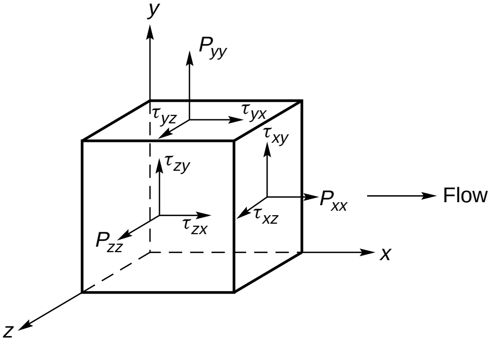

Figure 1.3 *Stress components in three dimensional flow*

normal component. Figure 1.3 shows the nine stress components schematically in an element of fluid. By considering the equilibrium of a fluid element, it can easily be shown that $\tau_{yx} = \tau_{xy}$; $\tau_{xz} = \tau_{zx}$ and $\tau_{yz} = \tau_{zy}$. The normal stresses can be visualised as being made up of two components: isotropic pressure and a contribution due to flow, i.e.

$$P_{xx} = -p + \tau_{xx} \tag{1.6a}$$

$$P_{yy} = -p + \tau_{yy} \tag{1.6b}$$

$$P_{zz} = -p + \tau_{zz} \tag{1.6c}$$

where τ_{xx}, τ_{yy}, τ_{zz}, contributions arising from flow, are known as deviatoric normal stresses for Newtonian fluids and as extra stresses for non-Newtonian fluids. For an incompressible Newtonian fluid, the isotropic pressure is given by

$$p = -\tfrac{1}{3}(P_{xx} + P_{yy} + P_{zz}) \tag{1.7}$$

From equations (1.6) and (1.7), it follows that

$$\tau_{xx} + \tau_{yy} + \tau_{zz} = 0 \tag{1.8}$$

For a Newtonian fluid in simple shearing motion, the deviatoric normal stress components are identically zero, i.e.

$$\tau_{xx} = \tau_{yy} = \tau_{zz} = 0 \tag{1.9}$$

Thus, the complete definition of a Newtonian fluid is that it not only possesses a constant viscosity but it also satisfies the condition of equation (1.9), or simply that it satisfies the complete Navier–Stokes equations. Thus, for instance, the so-called constant viscosity Boger fluids [Boger, 1976; Prilutski *et al*., 1983] which display constant shear viscosity but do not conform to equation (1.9) must be classed as non-Newtonian fluids.