

## Chapter Seven: Einstein special theory of relativity

### 7.1 Pre- relativistic physics

The starting point for our work is Newton's laws of motion. These can be stated as follows:

- *Free particles move with constant velocity.*
- *The vector force  $F$  is proportional to the rate of change of momentum i.e.*

$$F = \frac{d}{dt}(mv)$$

- *To every action there is an equal and opposite reaction.*

The first of these laws singles out inertial frames as the non- accelerating ones. Consider now a frame  $O$  [ i.e. a set of spatial coordinates  $(x, y, z)$  and a time coordinate  $t$  ], and another frame  $O'$  with coordinates  $(x', y', z', t')$  which moves in the  $x$  direction with uniform speed  $v$  relative to the frame  $O$ .

Common sense suggests that the two sets of coordinates are related by

$$\begin{aligned}x' &= x - vt, \\y' &= y, \\z' &= z, \\t' &= t\end{aligned}\tag{7.1}$$

These are the Galilean transformations.



Figure 7.1: I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me. *Isaac Newton.*

If the particle has a velocity ( $\mathbf{u}$ ) with components  $(u_1, u_2, u_3)$  in  $O$ , its velocity in  $O'$  is:

$$\begin{aligned}u'_1 &= u_1 - v, \\u'_2 &= u_2, \\u_3 &= u_3,\end{aligned}\tag{7.2}$$

where

$$u_1 = \frac{dx_1}{dt}, \quad u_2 = \frac{dx_2}{dt}, \quad u_3 = \frac{dx_3}{dt}\tag{7.3}$$

## 7.2 The equations of electromagnetism

Maxwell's equations for the electromagnetic field [in units with  $\epsilon_0 = \mu_0 = c = 1$ ] are:

$$\begin{aligned} \nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} &= 4\pi \vec{j}, & \nabla \times \vec{E} - \frac{\partial \vec{B}}{\partial t} &= 0, \\ \nabla \cdot \vec{E} &= 4\pi \rho, & \nabla \cdot \vec{B} &= 0 \end{aligned} \quad (7.4)$$

These equations show that one can have electromagnetic waves in a vacuum which travel with a speed  $c = 3 \times 10^8$  m/s, thus light is a form of electromagnetic radiation.



Figure 7.2: James Clerk Maxwell

Ether was postulated as the medium which carries these waves [i.e. the speed  $c$  is the speed with respect to the ether] and naturally this was identified with the absolute space of Newton.

Attempts to measure the ether drift by Michelson and Morley in 1887 gave a null result, so effects like stellar aberration could not be due to the earth dragging the ether.

Another great problem was that Maxwell's equations did not appear to obey the principle of Galilean Relativity i.e. they were not invariant under the Galilean transformations. This means that in a moving space ship the electric and optical phenomena should be different from those in a stationary ship!!! Thus one could use for example optical phenomena to determine the speed of the ship.

One of the consequences of Maxwell's equations is that if there is a disturbance in the field such that light is generated, these electromagnetic waves go out in all directions equally and at the same speed  $c$ . Another consequence of the equations is that if the source of the disturbance is moving, the light emitted goes through space at the same speed  $c$ . This is analogous to the case of sound being likewise independent of the motion of the source. This independence of the motion of the source in the case of light brings up an interesting problem.

Suppose we are riding in a car that is going at a speed  $u$ , and light from the rear is going past the car with speed  $c$ . According to the Galilean transformations, the apparent speed of the passing light, as we measure it in the car, should not be  $c$ , but  $c-u$ . This means that by measuring the speed of the light going past the car, one could determine the absolute speed of the car. This is crazy because we already know that the laws of mechanics do not permit the measurement of absolute velocity.

- Something is wrong with the laws of Physics!!

At first people thought that it was Maxwell's equations that were at fault and they modified them so that they were invariant under the Galilean

transformations; but this led to the prediction of new electrical phenomena which could not be found experimentally.

Then in 1903 Lorentz made a remarkable and curious discovery. He found that when he applied the following transformations to Maxwell's equations they remained invariant.

$$\begin{aligned}t' &= \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}, \\x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \\y' &= y \\z' &= z,\end{aligned}\tag{7.5}$$

These are the famous Lorentz transformations.

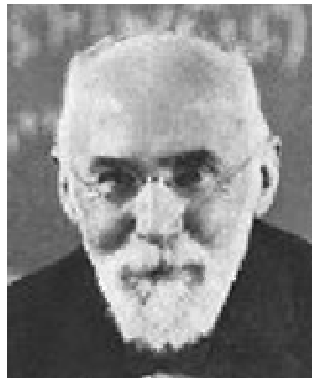


Figure 7.3: Hendrik Antoon Lorentz.

### 7.3 The principle of Special Relativity

In 1905 Einstein generalized the Galilean relativity principle [applicable only to Newton's laws] to the whole of Physics by postulating that:

1. *All inertial frames are equivalent for all experiments i.e. no experiment can measure absolute velocity.*
2. *Maxwell's equations and the speed of light must be the same for all observers.*

Einstein's motivation was to avoid inconsistencies between Maxwell's equations and Galilean relativity. The Lorentz transformations must relate to actual space and time measurements.

Special Relativity abolishes the idea of absolute space [for example the ether] and absolute time, but it leaves unexplained the origin of inertial frames. Mach's principle says that the inertial frames are determined by the rest of the matter in the universe [i.e. those that are non-accelerating with respect to the rest of the universe]. Einstein tried later to incorporate this idea into General Relativity.

### 7.4 Time dilation

Suppose we have someone in a spaceship  $O'$  moving at some velocity  $v$  relative to an observer  $O$ . The guy in the spaceship turns his stereo on and starts listening to some rap music!! With a clock he measures the time interval between two beats  $\Delta t' = t'_B - t'_A$ . What time interval does the observer  $O$  measure? To calculate this it is easier to use the inverse Lorentz transformations. By the Relativity principle we have:

$$t = \gamma(t' + vx'/c^2) \quad (7.6)$$

SO

$$t_B - t_A = \gamma \left[ t'_B - t'_A + \frac{v^2}{c^2} (x'_B - x'_A) \right] \quad (7.7)$$

But the stereo is stationary relative to  $O'$  so  $x'_A = x'_B$ , so we end up with:

$$\Delta t = t_B - t_A = \gamma \Delta t' = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (7.8)$$

Since  $\Delta t > \Delta t'$ , time in the spaceship slows down! That is, when the clock in the spaceship records 1 second elapsed, as seen by the man in the ship, it shows  $1/\sqrt{1 - v^2/c^2}$  seconds to the man outside.

The slowing of clocks in a moving system is a real effect and it applies equally to all kinds of time, for example biological, chemical reaction rates, even to the rate of growth of a cancer in a cancer patient! How is this so you may ask? If the rate of growth of the cancer was the same for a stationary patient as for a moving one, it would be possible to use the rate of cancer development to determine the speed of the ship!

A very interesting example of the slowing down of time is that of the cosmic ray muons. These are particles that disintegrate spontaneously after an average lifetime of about  $2.2 \times 10^{-6}$  seconds. It is clear that in its short lifetime a muon cannot, even at the speed of light, travel more than 600 m. But although the muons are created at the top of the atmosphere, some 10 km up, we can detect them down here on earth. How can that be!!? From the muon's point of view (i.e. from their frame of reference) they only live about  $2\mu\text{s}$ . However