١. Right- or Left-Linear Grammar:

A linear grammar is a grammar in which at most one variable can occur on the right/left side of any production without restriction on the position of this variable.

Definition (Right-linear):

A grammar G = (V,T, S, P) is said to be right-linear if all productions are of the form:

$A \rightarrow xB, \quad A \rightarrow x,$

where A, B∈V and x ∈ T*.

Definition (Left-linear):

A grammar G = (V,T, S, P) is said to be left-linear if all productions are of the form:

$A \rightarrow Bx,$

$A \rightarrow x,$

where A, B ∈ V and x ∈ T*.

Example ١:

Find L(G) where G = ({S, S١, S٢}, {a, b}, S, P) with

S → S١ab,
S١→S١ab | S٢,
S٢ → a.

Answer. This is a left-linear grammar.

S ⇒ S١ab ⇒ S١abab ⇒ S٢abab ⇒aabab.

Then

L(G) = {aabw |w ∈ (ab)*}.

٢. Hierarchy of Grammars(Chomsky Hierarchy):

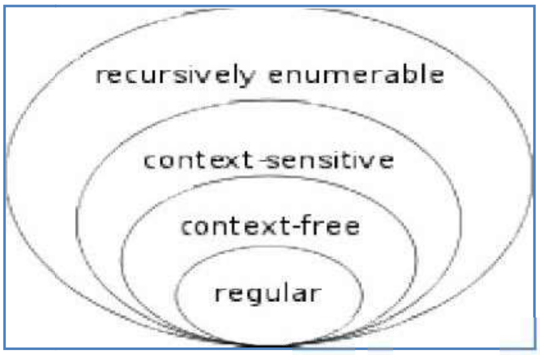
The Chomsky hierarchy classifies grammars according to syntactic restrictions on rules as following. Let G = (∑, V, S, P) be a grammar.

١. G is called a **Type-٠** grammar or an **unrestricted** grammar.

٢. G is a **Type-١** or **context-sensitive** grammar.

٣. G is a **Type-٢** or **context-free grammar**.

٤. G is a **Type-٣** or **regular** grammar.



٢.١ An Unrestricted Grammar:

A set of production rules of the form **α → β** where α and β are arbitrary strings of terminal and non-terminal symbols. The rules of these grammars do not have the restriction above, their left-hand sides may contain any string of terminal and /or non-terminal symbols, provided there is at least one non-terminal symbol. The type of automata which can recognize such a language is a Turing machine.

Example ٢:

L = {w ∈ {a, b, c}$^+$: number of a's, b's and c's is the same}

S → ABCS
S → ABC
AB → BA
BC → CB
AC → CA
BA → AB
CA → AC
CB → BC
A → a
B → b
C → c

٢.٢ A Context-Sensitive Grammar (CSG):

A production rules of the grammar have the form **α → β and |β| ≥|α|,** i.e. no production rule is length-decreasing.

- A language L is context-sensitive if it is generated by some context-sensitive grammar.

- Context-Sensitive grammars may have more than one symbol on the left-hand-side of their grammar rules, provided that at least one of them is a non-terminal and the number of symbols on the left-hand-side does not exceed the number of symbols on the right-hand-side.
- The automaton which recognizes a context-sensitive language is called a linear-bounded automaton.

An example٣(CSG) is:

S → aBCT|aBC
T → ABCT|ABC
BA → AB
CA → AC
CB → BC
aA → aa,
aB → ab
bB → bb,
bC → bc
cC →cc

Example ٤: The following grammar is context-sensitive.

S → aTb | ab

aT → aaTb | ac.

What is the language of the grammar?

{ab} U {a $^{n+١}$cb $^{n+١}$ | n ≥ ٠}. This language is context-free, it has the grammar

S → aTb | ab, and T → aTb | c. Any context-free language is context sensitive.

Exercise ١:

what is the language of the following grammar?

S → aBSc

S → aBc

Ba → aB

Bc → bc

Bb → bb

Exercise ٢: Let G be the grammar <N,∑ , P, S>, where N = {S},∑ = {a, b}, and P = {S→ϵ , S →aSbS}.

a. Find all the strings that are directly derivable from SaS in G.

b. Find all the derivations in G that start at S and end at ab.

c. Find all the sentential forms of G of length ٤ at most.

Exercise ٣: Find all the derivations of length ٣ at most that start at S in the grammar <N, ∑, P, S> whose production rules are:

S →AS

aS →bb

A →aa