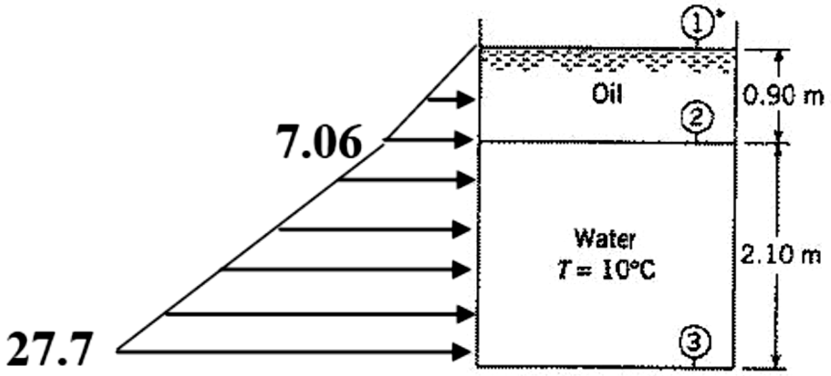
Problems on pressure variation and scales

Example 1: Oil with a specific gravity of 0.80 forms a layer 0.90 m deep in an open tank that is otherwise filled with water. The total depth of water and oil is 3 m. What is the gage pressure at the bottom of the tank?



$$p + \gamma z = \text{cons tan t}$$

$$p_1 + \gamma z_1 = p_2 + \gamma z_2$$

$$p_2 = p_1 + \gamma (z_1 - z_2)$$

$$p_1 = p_{atm} = 0$$

$$p_2 = \gamma_{oil} \Delta z = .8 \times 9810 \times .9 = 7.06\text{kPa}$$

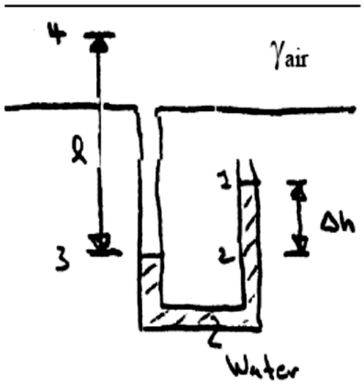
$$p_3 = p_2 + \gamma_{water} (z_2 - z_3)$$

$$= 7060 + 9810 \times 2.1$$

$$= 27.7\text{kPa}$$

Example:

Air at 20 °C is in pipe with a water manometer. For given conditions compute gage pressure in pipe.



l = 140 cm

Δh = 70 cm

p_4 = ? gage (i.e., $p_1 = 0$)

Pressure same at 2&3 since same elevation & Pascal's law: in closed system pressure change produce at one part transmitted throughout entire system

$\cancel{p_1} + \gamma \Delta h = p_3$ step-by-step method

$p_3 - \gamma_{air} l = p_4$

$\gamma \Delta h$

$p_1 + \gamma\Delta h - \gamma_{air} l = p_4$ complete circuit method

$\gamma\Delta h - \gamma_{air} l = p_4$ gage

$\gamma_{water}(20°C) = 9790\ N/m^3 \Rightarrow p_3 = \gamma\Delta h = 6853\ Pa\ [N/m^2]$

$\gamma_{air} = \rho g$

$$\rho = \frac{p}{RT} = \frac{\overset{p_{abs}}{\left(p_3 + p_{atm}\right)}}{\underset{°K}{R\left(°C + 273\right)}} = \frac{6853 + 101300}{287(20 + 273)} = 1.286\ kg/m^3$$

$\gamma_{air} = 1.286 \times 9.81 m/s^2 = 12.62\ N/m^3$

note $\gamma_{air} << \gamma_{water}$

$p_4 = p_3 - \gamma_{air} l = 6853 - \underbrace{12.62 \times 1.4}_{17.668} = 6835\ Pa$

if neglect effect of air column $\boxed{p_4 = 6853\ Pa}$

2.46 In Fig. P2.46 both ends of the manometer are open to the atmosphere. Estimate the specific gravity of fluid X.

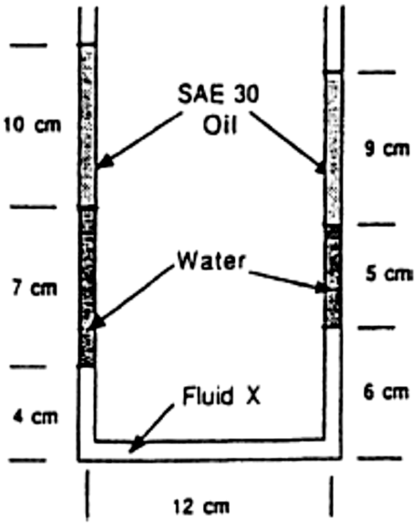


Fig. P2.46

Solution: The pressure at the bottom of the manometer must be the same regardless of which leg we approach through, left or right:

$$p_{atm} + (8720)(0.1) + (9790)(0.07) + \gamma_X(0.04) \quad \text{(left leg)}$$

$$= p_{atm} + (8720)(0.09) + (9790)(0.05) + \gamma_X(0.06) \quad \text{(right leg)}$$

$$\text{or:} \quad \gamma_X = 14150\ N/m^3, \quad SG_X = \frac{14150}{9790} \approx \mathbf{1.45} \quad \textit{Ans.}$$

2.41 The system in Fig. P2.41 is at 20°C. Determine the pressure at point A in pounds per square foot.

Solution: Take the specific weights of water and mercury from Table 2.1. Write the hydrostatic formula from point A to the water surface:

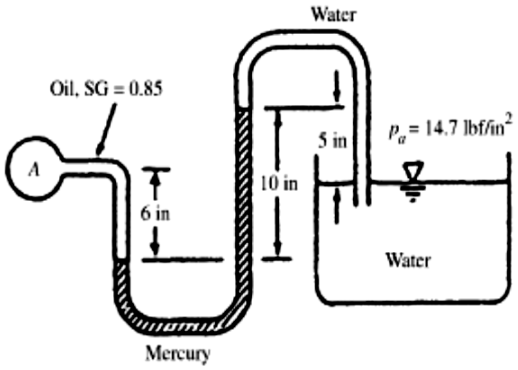


Fig. P2.41

$$p_A + (0.85)(62.4\ \text{lbf/ft}^3)\left(\frac{6}{12}\,\text{ft}\right) - (846)\left(\frac{10}{12}\right) + (62.4)\left(\frac{5}{12}\right) = p_{atm} = (14.7)(144)\,\frac{\text{lbf}}{\text{ft}^2}$$

$$\text{Solve for} \quad p_A = \mathbf{2770\ lbf/ft^2} \quad \textit{Ans.}$$

2.34 To show the effect of manometer dimensions, consider Fig. P2.34. The containers (a) and (b) are cylindrical and are such that $p_a = p_b$ as shown. Suppose the oil-water interface on the right moves up a distance $\Delta h < h$. Derive a formula for the difference $p_a - p_b$ when (a) $d \ll D$; and (b) $d = 0.15D$. What is the % difference?

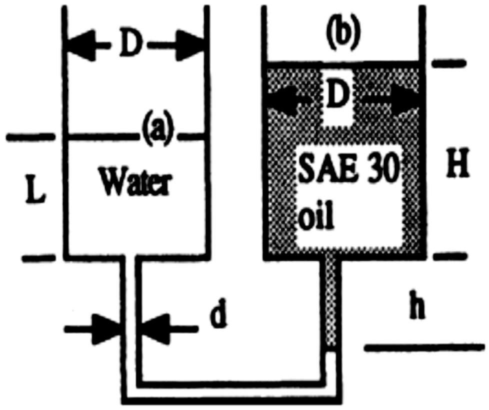


Fig. P2.34

Solution: Take $\gamma = 9790\ \text{N/m}^3$ for water and $8720\ \text{N/m}^3$ for SAE 30 oil. Let "H" be the height of the oil in reservoir (b). For the condition shown, $p_a = p_b$, therefore

$$\gamma_{water}(L + h) = \gamma_{oil}(H + h), \quad \text{or:} \quad H = (\gamma_{water}/\gamma_{oil})(L + h) - h \qquad (1)$$

Case (a), $d \ll D$: When the meniscus rises Δh, there will be no significant change in reservoir levels. Therefore we can write a simple hydrostatic relation from (a) to (b):

$$p_a + \gamma_{water}(L + h - \Delta h) - \gamma_{oil}(H + h - \Delta h) = p_b,$$

$$\text{or: } \mathbf{p_a - p_b = \Delta h(\gamma_{water} - \gamma_{oil})} \quad \textit{Ans.} \text{ (a)}$$

where we have used Eq. (1) above to eliminate H and L. Putting in numbers to compare later with part (b), we have $\Delta p = \Delta h(9790 - 8720) =$ *1070* Δh, with Δh in meters.

Case (b), d = 0.15D. Here we must account for reservoir volume changes. For a rise $\Delta h < h$, a volume $(\pi/4)d^2\Delta h$ of *water* leaves reservoir (a), decreasing "L" by $\Delta h(d/D)^2$, and an identical volume of oil enters reservoir (b), increasing "H" by the same amount $\Delta h(d/D)^2$. The hydrostatic relation between (a) and (b) becomes, for this case,

$$p_a + \gamma_{water}[L - \Delta h(d/D)^2 + h - \Delta h] - \gamma_{oil}[H + \Delta h(d/D)^2 + h - \Delta h] = p_b,$$

$$\text{or: } p_a - p_b = \mathbf{\Delta h[\gamma_{water}(1 + d^2/D^2) - \gamma_{oil}(1 - d^2/D^2)]} \quad \textit{Ans.} \text{ (b)}$$

where again we have used Eq. (1) to eliminate H and L. If d is not small, this is a *considerable* difference, with surprisingly large error. For the case d = 0.15 D, with water and oil, we obtain $\Delta p = \Delta h[1.0225(9790) - 0.9775(8720)] \approx 1486\ \Delta h$ or **39% more** than (a).

2.13 In Fig. P2.13 the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid?

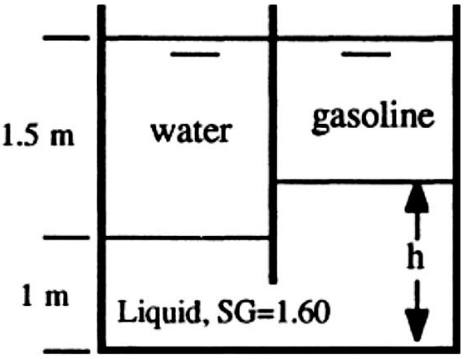


Fig. P2.13

Solution: Take water = 9790 N/m^3 and gasoline = 6670 N/m^3. The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

$$p_{bottom} = (9790\ \text{N/m}^3)(1.5\ \text{m}) + 1.60(9790)(1.0) = 1.60(9790)h + 6670(2.5 - h)$$

$$\text{Solve for } \quad h = \mathbf{1.52\ m} \quad \textit{Ans.}$$

Exercises:

1 Express standard atmospheric pressure as a head, $h = p/\rho g$, in (a) feet of ethylene glycol; (b) inches of mercury; (c) meters of water; and (d) mm of methanol.

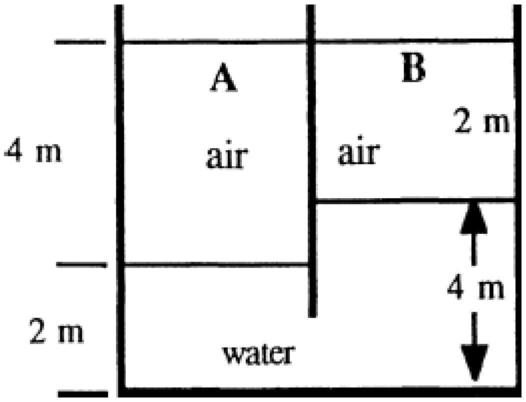
Ans: a- 30.3 ft; b- 30.0 in; c- 10.35 m; d- 13.13m.

2. A closed tank contains 1.5 m of SAE 30 oil, 1 m of water, 20 cm of mercury, and an air space on top, all at 20°C. If p_{bottom} = 60 kPa, what is the pressure in the air space?

Ans: 10500 Pa.

3. The closed tank in Fig. P2.14 is at 20°C. If the pressure at A is 95 kPa absolute, determine p at B (absolute). What percent error do you make by neglecting the specific weight of the air?

Ans:75420 Pa.



4. In Fig. P2.21 all fluids are at 20°C. Gage A reads 350 kPa absolute. Determine (a) the height h in cm; and (b) the reading of gage B in kPa absolute.

Ans: a- h= 6.49 m

b- P_B= 251kPa.