from our point of view they live considerably longer, indeed long enough to reach the surface of the earth (by a factor of $1/\sqrt{1-v^2/c^2}$).

7.6 Length contraction

Now suppose the observer *O* wants to measure the length of the spaceship. He can only do this by making an instantaneous measurement of the spatial coordinates of the end of the ship i.e. x_A and x_B.

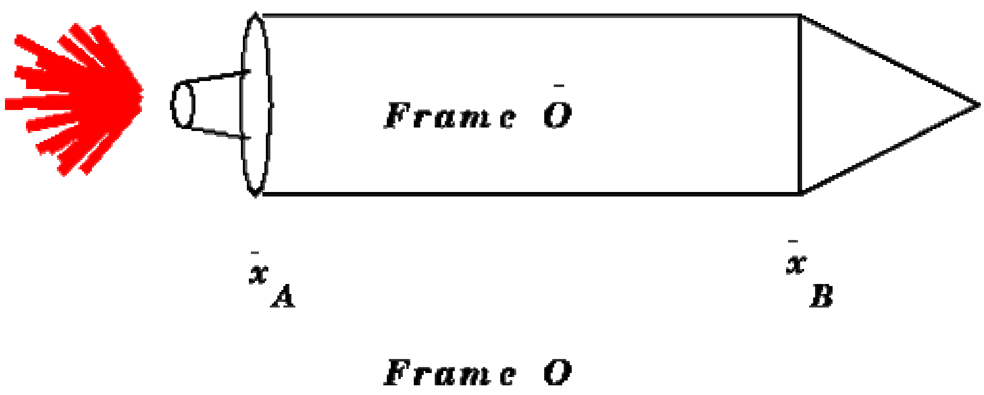

Figure 7.8: **Length contraction**

The Lorentz transformations give:

$$x_1' = \gamma(x_1 - vt_1)$$
$$x_2' = \gamma(x_2 - vt_2) \qquad \text{(7.9)}$$

so the length is

$$L = x_2 - x_1$$
$$= \gamma^{-1}(x_2' - x_1')$$
$$= \gamma^{-1}L' \qquad \text{(7.10)}$$

since $t_1 = t_2$ [instantaneous measurement by *O*]. Writing this result in terms of v and *c* we have:

$$L = \sqrt{1-v^2/c^2}\; L' \qquad \text{(7.11)}$$

Note that this is not a physical effect on the rod but an effect of space-time itself.

7.7 The twin paradox

To continue our discussion of the Lorentz transformations and relativistic effects, we consider the famous ``twin paradox'' of Peter and Paul. When they are old enough to drive a spaceship, Paul flies away from earth at very high speed. Because Peter, who is left on the ground, sees Paul going so fast, all of Pauls' clocks appear to go slower, from Peter's point of view. Of course, Paul notices nothing unusual. After a while he returns and finds that he is younger that Peter! Now some people might say ``heh, heh, heh, from the point of view of Paul, can't we say that Peter was moving and should therefore appear to age more slowly? By symmetry, the only possible result is that they are both the same age when they meet''. But in order for them to come back together and make a comparison, Paul must turn around which involves decelerating and accelerating and during that period he is not in an inertial frame. This breaks the apparent symmetry and so resolves the paradox.

Example: 1

(a) An observer on Earth sees a spaceship at an altitude of 4350 km moving downward toward Earth with a speed of 0.970*c*. What is the distance from the spaceship to Earth as measured by the spaceship's captain?

(b) After firing his engines, the captain measures her ship's altitude as 267 km, while the observer on Earth measures it to be 625 km. What is the speed of the spaceship at this instant?

Solution

(a) Find the distance from the ship to Earth as measured by the captain.

Substitute into Equation 7.11, getting the altitude as measured by the captain in the ship.

$$L = \sqrt{1-\frac{v^2}{c^2}}\ L' = \sqrt{1-\frac{(0.970c)^2}{c^2}}\ (4350\,\text{km}) = 1.06\times10^3\ \text{km}$$

(b) What is the subsequent speed of the spaceship if the Earth observer measures the distance from the ship to Earth as 625 km and the captain measures it as 267 km?

$$L = \sqrt{1-\frac{v^2}{c^2}}\ \overline{L} \Rightarrow \quad L^2 = \left(1-\frac{v^2}{c^2}\right)\overline{L}^2 \Rightarrow 1-\frac{v^2}{c^2} = \left(\frac{L}{\overline{L}}\right)^2$$

$$v = c\sqrt{1-\left(L/\overline{L}\right)^2} = c\sqrt{1-\left(267\text{km}/625\text{km}\right)^2} = 0.904\,c$$