Turbulent boundary layers

The treatment of turbulent boundary layers is far more difficult due to the time-dependent variation of the flow properties. One of the most widely used techniques in which turbulent flows are tackled is to apply Reynolds decomposition. Here the instantaneous flow properties are decomposed into a mean and fluctuating component. Applying this technique to the boundary layer equations gives the full turbulent boundary layer equations not often given in literature:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0$$

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \nu\left(\frac{\partial^2 \overline{u}}{\partial x^2} + \frac{\partial^2 \overline{u}}{\partial y^2}\right) - \frac{\partial}{\partial y}(\overline{u'v'}) - \frac{\partial}{\partial x}(\overline{u'^2})$$

$$\overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial y} + \nu\left(\frac{\partial^2 \overline{v}}{\partial x^2} + \frac{\partial^2 \overline{v}}{\partial y^2}\right) - \frac{\partial}{\partial x}(\overline{u'v'}) - \frac{\partial}{\partial y}(\overline{v'^2})$$

Using the same order-of-magnitude analysis as for the instantaneous equations, these turbulent boundary layer equations generally reduce to become in their classical form:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0$$

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \nu\frac{\partial^2 \overline{u}}{\partial y^2} - \frac{\partial}{\partial y}(\overline{u'v'})$$

$$\frac{\partial \overline{p}}{\partial y} = 0$$

The additional term $\overline{u'v'}$ in the turbulent boundary layer equations is known as the Reynolds shear stress and is unknown a priori. The solution of the turbulent boundary layer equations therefore necessitates the use of a turbulence model, which aims to express the Reynolds shear stress in terms of known flow variables or derivatives. The lack of accuracy and generality of such models is a major obstacle in the successful prediction of turbulent flow properties in modern fluid dynamics.

A laminar sub-layer exists in the turbulent zone; it occurs due to those fluid molecules which are still in the very proximity of the surface, where the shear stress is maximum and the velocity of fluid molecules is zero.

Heat and mass transfer

In 1928, the French engineer André Lévêque observed that convective heat transfer in a flowing fluid is affected only by the velocity values very close to the surface.[1][2] For flows of large Prandtl number, the temperature/mass transition from surface to freestream temperature takes place across a very thin region close to the surface. Therefore, the most important fluid velocities are those inside this very thin region in

which the change in velocity can be considered linear with normal distance from the surface. In this way, for

$$u(y) = u_0 \left[1 - \frac{(y-h)^2}{h^2}\right] = u_0 \frac{y}{h}\left[2 - \frac{y}{h}\right] \ ,$$

when $y \to 0$, then

$$u(y) \approx 2u_0 \frac{y}{h} = \theta y \ ,$$

where θ is the tangent of the Poiseuille parabola intersecting the wall. Although Lévêque's solution was specific to heat transfer into a Poiseuille flow, his insight helped lead other scientists to an exact solution of the thermal boundary-layer problem.[3] Schuh observed that in a boundary-layer, *u* is again a linear function of *y*, but that in this case, the wall tangent is a function of *x*.[4] He expressed this with a modified version of Lévêque's profile,

$$u(y) = \theta(x)y \ .$$

This results in a very good approximation, even for low Pr numbers, so that only liquid metals with Pr much less than 1 cannot be treated this way.[3] In 1962, Kestin and Persen published a paper describing solutions for heat transfer when the thermal boundary layer is contained entirely within the momentum layer and for various wall temperature distributions.[5] For the problem of a flat plate with a temperature jump at $x = x_0$, they propose a substitution that reduces the parabolic thermal boundary-layer equation to an ordinary differential equation. The solution to this equation, the temperature at any point in the fluid, can be expressed as an incomplete gamma function.[2] Schlichting proposed an equivalent substitution that reduces the thermal boundary-layer equation to an ordinary differential equation whose solution is the same incomplete gamma function.[6]