

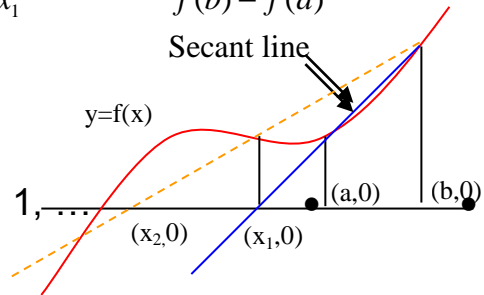
**Secant Method:**

Suppose a continuous function  $f$  defined on the interval  $[a,b]$  is given, but now we do not force the iterates to bracket a root. The graph of the function  $f$  is approximated by a secant line, we get

$$\frac{f(b) - f(a)}{b - a} = \frac{f(b) - y}{b - x} \Rightarrow \frac{f(b) - f(a)}{b - a} = \frac{f(b) - 0}{b - x_1} \Rightarrow x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Similarly  $x_2 = \frac{bf(x_1) - x_1f(b)}{f(x_1) - f(b)}$

In general  $x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}; i=0, 1, \dots$



$x_{i+2}$  is an approximate root if  $|x_{i+2} - x_{i+1}| < \varepsilon$  for any  $i$ .

Ex:

Find the root of the function  $x^3 + 4x^2 - 10$  by using Secant method with  $\varepsilon = 1.3 \times 10^{-8}$

Solution :

Let  $x_0 = 1$                        $x_1 = 2$

$f(x_0 = 1) = -5$                        $f(x_1 = 2) = 14$

$$x_2 = x_1 - \frac{f(x_1) * (x_1 - x_0)}{f(x_1) - f(x_0)} = 2 - \frac{14 * (2 - 1)}{14 - (-5)} = \frac{24}{19} = 1.2631$$

$$\Rightarrow f(x_2) = -1.602274379$$

Continue to compute  $x_3 = 1.338827839$  where

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

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$$|x_7 - x_6| = |1.3652300134 - 1.3652300011| < 1.3 \times 10^{-8}$$

n	$x_n$	$f(x_n)$
0	1	-5
1	2	14
2	1.2631578947	-1.6022743840
3	1.3388278388	-0.4303647480
4	1.3666163947	0.0229094308
5	1.3652119026	-0.002990679
6	1.3652300011	-0.0000002032

Note:

In method of false Position on the interval  $[a_n, b_n]$

$$x_{n+1} = a_n - \frac{f(a_n)(b_n - a_n)}{f(b_n) - f(a_n)} \text{ while in Secant method}$$

$$x_{n+1} = b_n - \frac{f(b_n)(b_n - a_n)}{f(b_n) - f(a_n)} \text{ in the same interval .}$$