Solved Problems and Questions on fluid properties

1. The quantities viscosity $\mu$, velocity $V$, and surface tension $Y$ may be combined into a dimensionless group. Find the combination which is proportional to $\mu$. This group has a customary name, which begins with $C$. Can you guess its name?

Solution: The dimensions of these variables are $\{\mu\} = \{M/LT\}$, $\{V\} = \{L/T\}$, and $\{Y\} = \{M/T^2\}$. We must divide $\mu$ by $Y$ to cancel mass $\{M\}$, then work the velocity into the group:

\[
\frac{\mu}{Y} = \left(\frac{M/LT}{M/T^2}\right) = \left(\frac{L}{T}\right).
\]

hence multiply by $\{V\} = \left(\frac{T}{L}\right)$

finally obtain Ans:

\[
\mu \frac{V}{Y} = \text{dimensionless}.
\]

This dimensionless parameter is commonly called the Capillary Number.

2. In the Fig., if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at $V = 5.5$ m/s? Note that glycerin viscosity $= 1.5$ N · s/m².

Solution: The shear stress is found from Eq:

\[
\tau = \mu \frac{\text{d}u}{\text{d}y} = \mu \frac{V}{h}
\]

\[
\tau = \frac{\mu V}{h} = \frac{1.5 \text{ Pa.s} \times 5.5 \text{m/s}}{0.006 \text{m}} = 1380 \text{ Pa}
\]

3. Suppose that the fluid being sheared in Fig. 1.6 is SAE 30 oil at 20°C. Compute the shear stress in the oil if $V = 3$ m/s and $h = 2$ cm.

The shear stress is found from Eq:
From Table for SAE 30 oil, \( \mu = 0.29 \text{ kg/(m . s)} \). Then, for the given values of \( V \) and \( h \), Eq. (1) predicts:
\[
\tau = \frac{\mu \frac{dv}{dt}}{h} = \frac{\mu V}{h}
\]

4. When a vehicle such as an automobile slams on its brakes (locking the wheels) on a very wet road it can “hydroplane.” In these circumstances a film of water is created between the tires and the road. Theoretically, a vehicle could slide a very long way under these conditions though in practice the film is destroyed before such distances are achieved (indeed, tire treads are designed to prevent the persistence of such films). To analyze this situation, consider a vehicle of mass, \( M \), sliding over a horizontal plane covered with a film of liquid of viscosity, \( \mu \). Let the area of the film under all four tires be \( A \) and the film thickness (assumed uniform) be \( h \).

a. If the velocity of the vehicle at some instant is \( V \), find the force slowing the vehicle down in terms of \( A \), \( V \), \( h \), and \( \mu \).

b. Find the distance, \( L \), that the vehicle would slide before coming to rest assuming that \( A \) and \( h \) remain constant (this is not, of course, very realistic).

c. What is this distance, \( L \), for a 1000 kg vehicle if \( A = 0.1 \text{ m}^2 \), \( h = 0.1 \text{ mm} \), \( V = 10 \text{ m/s} \), and the water viscosity is \( \mu = 0.001 \text{ kg/(m·s)} \)?

Sol:

a. \[
\tau = \frac{F}{A} = \frac{\mu V}{h} \]
\[
\therefore F = \frac{\mu V}{h} A
\]

b. \[
L = \frac{MhV_0}{\mu A}
\]

c. \( L = 10,000 \text{ m} \)

5. The specific weight of water at ordinary pressure and temperature is 62.4 lb/ft\(^3\). The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution:
\[
\rho_{\text{water}} = \frac{\gamma_{\text{water}}}{g} = 1.938 \text{ slugs/ft}^3
\]
\[
\gamma_{\text{mercury}} = S.G. \text{mercury} \times \gamma_{\text{water}} = 846 \text{ lb/ft}^3
\]
\[
\rho_{\text{mercury}} = S.G. \text{mercury} \times \rho_{\text{water}} = 26.3 \text{ slugs/ft}^3
\]

6. The specific weight of water at ordinary pressure and temperature is 9.81 kN/m\(^3\). The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Sol:

Ans: a. 1000 kg/m\(^3\)
    b. 133.0 kN/m\(^3\)
    c. 1356 kg/m\(^3\)

EXERCISES
1. If the specific weight of a liquid is 52 lb/ft$^3$, what is its density? Ans: 1.616 slug/ft$^3$
2. If the specific weight of a liquid is 8.1 kN/m$^3$, what is its density? Ans: 826 kg/m$^3$
3. If the specific volume of a gas is 375 ft$^3$/slug, what is its specific weight in lb/ft$^3$? Ans: 0.0858 lb/ft$^3$
4. If the specific volume of a gas is 0.70 m$^3$/kg, what is its specific weight in N/m$^3$? Ans: 14 N/m$^3$
5. A certain gas weighs 16.0 N/m$^3$ at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 12.0 N/m$^3$? Ans: 1.63 kg/m$^3$, 0.613 m$^3$/kg, 1.33.
6. The specific weight of glycerin is 78.6 lb/ft$^3$. Compute its density and specific gravity. What is its specific weight in kN/m$^3$? 2.44 slug/ft$^3$, 1.26, 40.51KN/m$^3$
7. If a certain gasoline weighs 43 lb/ft$^3$, what are the values of its density, specific volume, and specific gravity relative to water at 60°F? 19.79 slug/ft$^3$, 0.05 ft$^3$/slug, 0.69
8. A rigid cylinder, inside diameter 15 mm, contains a column of water 500 mm long. What will the column length be if a force of 2 kN is applied to its end by a frictionless plunger? Assume no leakage. Ans: 499.974 mm
9. A flat plate 200 mm 750 mm slides on oil ($\mu = 0.85$ Ns/m$^2$) over a large plane surface. What force $F$ is required to drag the plate at a velocity $v$ of 1.2 m/s, if the thickness $t$ of the separating oil film is 0.6 mm? Ans: 11333.33 N.
10. A space 16 mm wide between two large plane surfaces is filled with SAE 30 Western lubricating oil at 35°C. What force $F$ is required to drag a very thin plate of 0.4 m$^2$ area between the surfaces at a speed $v$ 0.25 m/s (a) if the plate is equally spaced between the two surfaces, and (b) if t 5 mm? Ans: a-7.25 N, b- 8.436 N
11. Distilled water at 20°C stands in a glass tube of 6.0-mm diameter at a height of 18.0 mm. What is the true static height? Ans: 12.969 mm.
12. Compute the capillary depression of mercury at 68°F ($\Theta = 140^\circ$) to be expected in a 0.05-in-diameter tube. Ans: 3.65*10$^{-4}$ in.
13. Compute the capillary rise in mm of pure water at 10°C expected in an 0.8-mm diameter tube. Ans: 0.037 mm