2.nd week

COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS

Fluid mechanics deals with both incompressible and compressible fluids, that is, with liquids and gases of either constant or variable density. Although there is no such thing in reality as an incompressible fluid, we use this term where the change in density with pressure is so small as to be negligible. This is usually the case with liquids. We may also consider gases to be incompressible when the pressure variation is small compared with the absolute pressure. Ordinarily we consider liquids to be incompressible fluids, yet sound waves, which are really pressure waves, travel through them. This is evidence of the elasticity of liquids. In problems involving water hammer we must consider the compressibility of the liquid. The flow of air in a ventilating system is a case where we may treat a gas as incompressible, for the pressure variation is so small that the change in density is of no importance. But for a gas or steam flowing at high velocity through a long pipeline, the drop in pressure may be so great that we cannot ignore the change in density. For an airplane flying at speeds below 250 mph (100 m/s), we may consider the air to be of constant density. But as an object moving through the air approaches the velocity of sound, which is of the order of 760 mph (1200 km/h) depending on temperature, the pressure and density of the air adjacent to the body become materially different from those of the air at some distance away, and we must then treat the air as a compressible fluid.

COMPRESSIBILITY OF LIQUIDS The compressibility (change in volume due to change in pressure) of a liquid is inversely proportional to its volume modulus of elasticity, also known as the bulk modulus. This modulus is defined as

$$\boldsymbol{E_v = -v\frac{dp}{dv} = -\left(\frac{v}{dv}\right)dp}$$

where v specific volume and p pressure. As v/dv is a dimensionless ratio, the units of Ev and p are identical. The bulk modulus is analogous to the modulus of elasticity for solids; however, for fluids it is defined on a volume basis rather than in terms of the familiar one-dimensional stress–strain relation for solid bodies.

Note that at any one temperature the bulk modulus of water does not vary a great deal for a moderate range in pressure. By rearranging the definition of Ev, as an approximation we may use for the case of a fixed mass of liquid at constant temperature**:**

$$\boldsymbol{\frac{\Delta v}{v} = -\frac{dp}{E_v} \quad \text{or} \quad \frac{v_2 - v_1}{v} = -\frac{p_2 - p_1}{E_v}}$$

where Ev is the mean value of the modulus for the pressure range and the subscripts 1 and 2 refer to the before and after conditions. Assuming Ev to have a value of 320,000 psi, we see that increasing the pressure of water by 1000 psi will compress it only 0.3%, of its original justified.

Example:

At a depth of 8 km in the ocean the pressure is 81.8 MPa. Assume that the specific weight of seawater at the surface is 10.05 kN/m3 and that the average volume modulus is 2.34 109 N/m2 for that pressure range. (a) What will be the change in specific volume between that at the surface and at that depth? (b) What will be the specific volume at that depth? (c) What will be the specific weight at that depth?
Sol:

a. $v_1 = \frac{1}{\rho_1} = \frac{g}{\gamma_1} = \frac{9.81}{10050}$= 0.000976 m^3/kg

   Δv = -0.000976(81.8 * 10 $^{-6}$ - 0)/(2.34 * 10^9)= -34.1 10^{-6} m^3 /kg

b. $v_2 = v_1 + \Delta v = 0.00942$ m^3/kg

c. $\gamma_2 = \frac{g}{v_2}$ = 9.81/0.000942 = 10410 N/m^3

SURFACE TENSION :

Liquids have cohesion and adhesion, both of which are forms of molecular attraction. Cohesion enables a liquid to resist tensile stress, while adhesion enables it to adhere to another body. At the interface between a liquid and a gas, i.e., at the liquid surface, and at the interface between two immiscible (not mixable) liquids, the out-of-balance attraction force between molecules forms an imaginary surface film which exerts a tension force in the surface. This liquid property is known as surface tension. Because this tension acts in a surface, we compare such forces by measuring the tension force per unit length of surface. When a second fluid is not specified at the interface, it is understood that the liquid surface is in contact with air. The surface tensions of various liquids cover a wide range, and they decrease slightly with increasing temperature. Values of the surface tension for water between the freezing and boiling points vary from 0.00518 to 0.00404 lb/ft (0.0756 to 0.0589 N/m). Capillarity is the property of exerting forces on fluids by fine tubes or porous media; it is due to both cohesion and adhesion. When the cohesion is of less effect than the adhesion, the liquid will wet a solid surface it touches and rise at the point of contact; if cohesion predominates, the liquid surface will depress at the point of contact. For example, capillarity makes water rise in a glass tube, while mercury depresses below the true level, as shown in the insert in Fig. 2.7, which is drawn to scale and reproduced actual size. We call the curved liquid surface that develops in a tube a meniscus. A cross section through capillary rise in a tube looks like Fig. 2.8. From free-body considerations, equating the lifting force created by surface tension to the gravity force,

$$2\pi r \sigma cos\theta = \pi r^2 h \gamma$$

$$h = \frac{2\sigma cos\theta}{\gamma r}$$

where σ=surface tension (sigma) in units of force per unit length

θ= wetting angle (theta)

γ= specific weight of liquid

r= radius of tube
h= capillary rise
Note: For tube diameters larger than in (12 mm), capillary effects are negligible
Surface tension effects are generally negligible in most engineering situations. However, they can be important in problems involving capillary rise, such as in the soil water zone; without capillarity most forms of vegetable life would perish. When we use small tubes to measure fluid properties, such as pressures, we must take the readings while aware of the surface tension effects; a true reading would occur if surface tension effects were zero. These effects are also important in hydraulic model studies when the model is small, in the breakup of liquid jets, and in the formation of drops and bubbles. The formation of drops is extremely complex to analyze, but is, for example, of critical concern in the design of inkjet printers,

Example : Water at 10°C stands in a clean glass tube of 2-mm diameter at a height of 35 mm. What is the true static height?

at 10°C: ɣ= 9804 N/m^3 , б= 0.0742 N/m. for clean glass tube: θ=-0°.

$$h = \frac{2\sigma cos\theta}{\gamma r} = 0.015\ 14 \text{ m} = 15.14 \text{ mm}$$

True static height= 35.00 - 15.14 =19.86 mm