Computing Derivatives

We can compute symbolic derivatives using MATLAB with a call to the *diff* command. Simply pass the function you want to differentiate to diff as this example shows:

```
>> syms x t
>> f = x^2;
>> g = sin(10*t);
>> diff(f)

ans =

2*x


>> diff(g)

ans =

10*cos(10*t)
```

To take higher derivatives of a function *f*, we use the syntax diff(*f*,*n*). Let's find the second derivative of $t \exp(-3t)$:

```
>> f = t*exp(-3*t);
>> diff(f,2)

ans =

-6*exp(-3*t)+9*t*exp(-3*t)
```

As you can see, diff returns the result of calculating the derivative—so we can assign the result to another variable that can be used later.

EXAMPLE 6-3
Show that $f(x) = x^2$ satisfies $-\frac{df}{dx} + 2x = 0$.

SOLUTION 6-3
We start by making some definitions:

```
>> syms x
>> f = x^2; g = 2*x;
```

Now let's compute the required derivative:

```
>> h = diff(f);
```

Finally, verify that the relation is satisfied:

```
>> -h+g

ans =

0
```

EXAMPLE 6-4
Does $y(t) = 3 \sin t + 7 \cos 5t$ solve $y'' + y = -5 \cos 2t$?

SOLUTION 6-4
Define our function:

```
>> y = 3*sin(t)+7*cos(5*t);
```

Now let's create a variable to hold the required result:

```
>> f = -5*cos(2*t);
```

To enter the left-hand side of the differential equation, we create another variable:

```
>> a = diff(y,2)+y;
```

We use isequal to check whether the equation is satisfied:

```
>> isequal(a,f)

ans =

     0
```

Since 0 is returned, $y(t) = 3 \sin t + 7 \cos 5t$ does not solve $y'' + y = -5 \cos 2t$.

EXAMPLE 6-5
Find the minima and maxima of the function $f(x) = x^3 - 3x^2 + 3x$ in the interval [0, 2].

SOLUTION 6-5
First let's enter the function and plot it over the given interval:

```
>> syms x
>> f = x^3-3*x^2+3*x;
>> ezplot(f, [0 2])
```

The plot is shown in Figure 6-4. To find local maxima and minima, we will compute the derivative and find the points at which it vanishes.

The derivative is:

```
>> g = diff(f)

g =

3*x^2-6*x+3
```

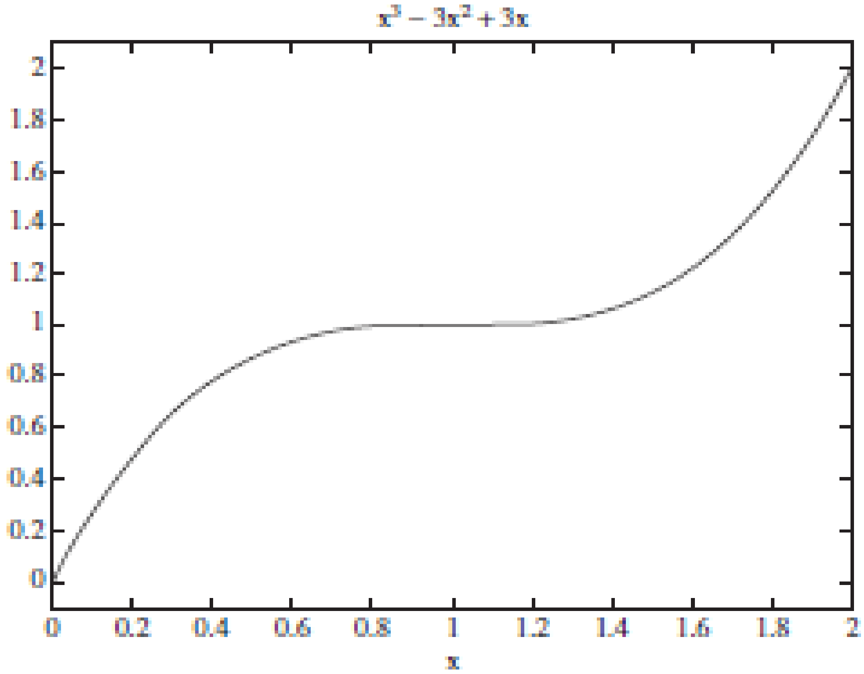

Figure 6-4 A plot of $f(x) = x^3 - 3x^2 + 3x$

As a quick aside, we can use the pretty command to make our expressions look nicer:

```
>> pretty(g)

                                        2
                                     3 x  - 6 x + 3
```

Well slightly nicer, anyway. Returning to the problem, let's set the derivative equal to zero and find the roots:

```
>> s = solve(g)

s =

 1
 1
```

We see that there is only one critical point, since the derivative has a double root. We can see from the plot that the maximum occurs at the endpoint, but let's prove this by evaluating the function at the critical points $x = 0, 1, 2$.

We can substitute a value in a symbolic function by using the *subs* command. With a single variable this is pretty simple. If we want to set $x = c$, we make the call subs(f,c). So let's check f for $x = 0, 1, 2$. We can check all three on a single line and have MATLAB report the output by passing a comma-delimited list:

```
>> subs(f,0), subs(f,1), subs(f,2)

ans =

     0


ans =

     1


ans =

     2
```

Since $f(2)$ returns the largest value, we conclude that the maximum occurs at $x = 0$. For fun, let's evaluate the derivative at these three points and plot it:

```
>> subs(g,0),subs(g,1),subs(g,2)

ans =

     3


ans =

     0


ans =

     3
```

Where are the critical points of the derivative? We take the second derivative and set equal to zero:

```
>> h = diff(g)

h =
```

```
6*x-6

>> solve(h)

ans =

1
```

The next derivative is:

```
>> y = diff(h)

y =

6
```

Since $g'' > 0$ we can conclude that the critical point $x = 1$ is a local minimum. A plot of $g(x)$ is shown in Figure 6-5.

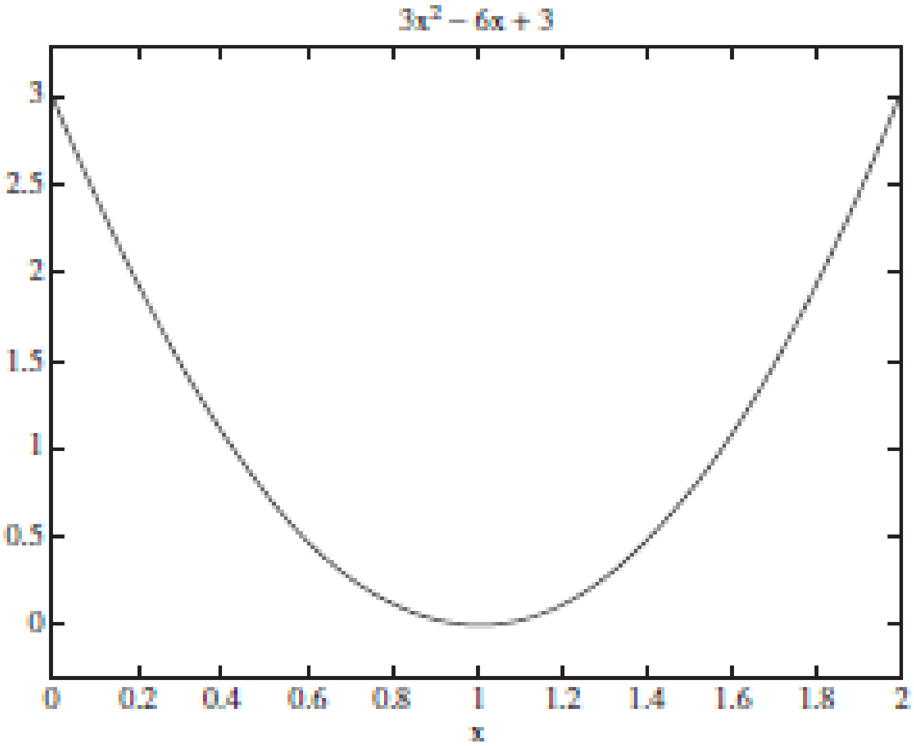

Figure 6-5 A plot of $g(x)$ showing the minimum we found in Example 6-5

EXAMPLE 6-6

Plot the function $f(x) = x^4 - 2x^3$ and show any local minima and maxima.

SOLUTION 6-6

First we define the function and plot it:

```
>> clear
>> syms x
>> f = x^4-2*x^3;
>> ezplot(f,[-2 3])
>> hold on
```

Now compute the first derivative:

```
>> g = diff(f)

g =

4*x^3-6*x^2
```

We find the critical numbers by setting the first derivative equal to zero and solving:

```
>> s = solve(g)

s =

 3/2
   0
   0
```

Next we compute the second derivative:

```
>> h = diff(g)

h =

12*x^2-12*x
```

Evaluating at the first critical number which is $x = 3/2$ we find:

```
>> a = subs(h,s(1))

a =

9
```

Since $f''(3/2) = 9 > 0$, the point $x = 3/2$ is a local minimum. Now let's check the other critical number:

```
>> b = subs(h,s(2))

b =

0
```

In this case, $f''(0) = 0$, which means that we cannot get any information about the point using the second derivative test. It can be shown easily that the point is neither a minimum nor a maximum. Now let's fix up the plot to show the local minimum. We add the point $(c, f(c))$ where $c = s(1)$ to the plot using the following command:

```
>> plot(double(s(1)),double(subs(f,s(1))),'ro
```

This puts a small red circle at the point $(3/2, f(3/2))$ on the plot. We told MATLAB to make the circle red by entering 'ro' instead of 'o', which would have added a black circle. Now let's label the point as a local minimum. This can be done using the text command. When you call text, you pass it the x-y coordinates where you want the text to start printing, and then pass the text string you want to appear on the plot:

```
>> text(0.8,3.2,'Local minimum')
>> hold off
```

The result is shown in Figure 6-6.

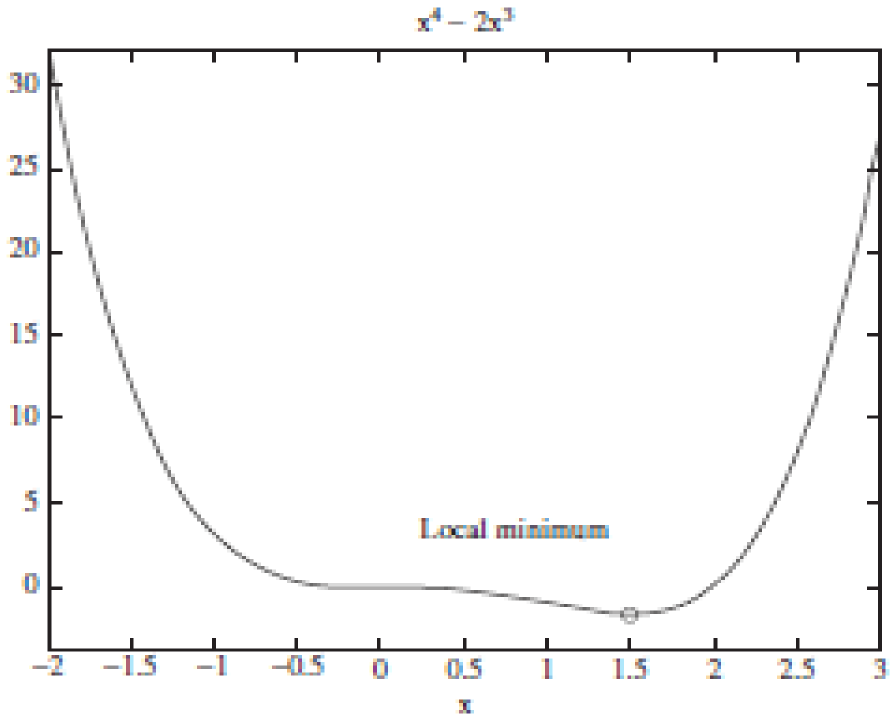

Figure 6-6 A plot of $f(x) = x^4 - 2x^3$ identifying the local minimum found in Example 6-6