$chain\,rule\,of\,\,the\,derivative:-$

$Statement\ :-$

$suppose\ f\ is\ differentiable\ at\ z_0\ and\ g\ is\ differentiable\ at\ f(z_0)\ then\ gof$

$is\ differentiable\ at\ z_0\ .and\ (gof\acute{)}(z_0) = \acute{g}\big(f(z_0)\big).\acute{f}(z_0)$

$example:-$

$find\ out\ the\ derivative\ of\ (z^3 - 2z^2 + iz)^7$

$solution:$

$taking\ the\ help\ of\ chain\ rule\ me\ got$

$7.(z^3 - 2z^2 + iz)^6.(3z^2 - 4z + i)$

$Cauchy - Riemann\ equation:-$

$statement:-$

$suppose\ f(z) = u + iv\ is\ differentiable\ at\ z = (x,y), then\ at\ (x,y)$

$the\ first\ order\ partial\ derivative\ of\ u\ and\ v\ must$

$exist\ , u_x = v_y\ and\ u_y = -v_x\ must\ be\ true\ at\ (x,y), further.$

$\acute{f}(z) = u_x + iv_x\ , where\ the\ partial\ derivative\ are\ evaluated\ at\ (x,y).$

$these\ partial\ differentiable\ equation$

$i.e\ u_x = v_y\ and\ u_y = -v_x\ are\ called\ cauchy - Riemann\ equation$

$note:-$

$this\ theorem\ tells\ that\ for\ the\ differentiablilty\ of\ f\ at$

$$z = (x,y)\ \ , \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0\ and \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

must be satisfied.

$example: -$

$1. f(z) = x^2 - y^2 + i2xy$

$solution: -$

$in\ this\ case$

$u(x, y) = x^2 - y^2$

$v(x, y) = 2xy$

$then\ u_x = 2x \qquad , v_x = 2y$

$u_y = -2y \qquad , v_y = 2x$

$so\ u_x = v_y \quad and\ u_y = -v_x \ \ at\ each\ \ (x, y) \in \mathbb{C}, since\ by\ the\ previouse\ result$

$f'(z) = u_x + iv_x = v_y - iu_y$

$= 2x + i2y$

$= 2(x + iy) = 2z$

$equavelently, f(z) = z^2\ means\ \acute{f}(z) = 2z$

$2. suppose\ f(z) = |z|^2 = x^2 + y^2 + i.0$

$solution:$

$u(x, y) = x^2 + y^2$

$v(x, y) = 0$

$while$

$u_x = 2x \qquad , u_y = 2y$

$v_x = 0 \qquad , v_y = 0$

$note \quad that\ C.R. equation$

$i.e.\ u_x = v_y \ , u_y = -v_x \ \ are\ true\ only\ at\ (0,0)\ further, \acute{f}(z)\ at\ (0,0)\ is$

$\acute{f}(z) = u_x + iv_x = 0 \ \ at\ (0,0)$

$3 - f(z) = -\sin x \cos hy - i \cos x \sin hy$

$so\ for\ continuity\ \&\ differentiability$

$here\ u(x,y) = -sinx\ coshy$

$v(x,y) = -cosx\ sinhy$

$u_x = -cosx\ coshy \quad , v_x = sinx \sin hy$

$u_y = -sinx\ sinhy \quad , v_y = -cosx\ coshy$

$u_x = v_y$

$u_y = -v_x$

$\acute{f}(z) = u_x + iv_x = -cosx\ coshy + isinx \sin hy$

H.w

Applying Cauchy-Riemann equation

1-f(z)=3x+y+i(3y-x)

2- f(z)=sinx coshy+icosx sinhy

3- f(z)=$e^{-y}\ sinx - ie^{-y} cosx$

4-f(z)=(z^2-2) $e^{-x}e^{-y}$