Solutions of Equations in one Variable.

Locating the position of roots (programming method):

To locate the position of roots of the function (equation) f(x)=0 by using ***programming method,*** let f(x) be continuous function on the interval [a,b]. We divide the interval [a,b] into n subintervals $a=x_0<x_1<\ldots<x_{n-1}<x_n=b$ where $x_i=a+ih$, i=0, 1, … ,n ; $h=\frac{b-a}{n}$. If $f(x_i)\times f(x_{i+1})<0$ for any $0\le i\le n$, then there exits c, a<c<b for which f(c)=0.

Example 1:

Find the approximate location of the function

1. $f(x)=x^4-7x^3+3x^2+26x-10=0$ on the interval [-8,8] with n=4 and n=8..
2. $f(x)=x^3+4x^2-10=0$ on the interval [1,2] with n=5.
3. $f(x)=x^3-4x+1=0$ on the interval [-1,4] with n=5.

Solution: (1): Let n=4, $h=\frac{b-a}{n}=\frac{8-(-8)}{4}=4$

x	-8	-4	0	4	8
f(x)	+	+	-	-	+

There is a root between (-4,0) and (4,8).

If n=8, h=2:

x	-8	-6	-4	-2	0	2	4	6	8
f(x)	+	+	+	+	-	+	-	+	+

There is a root between (-2,0), (0,2), (2,4) and (4,6).

Solution: (2): Let n=5, h=0.2

x	1	1.2	1.4	1.6	1.8	2
f(x)	-	-	+	+	+	+

There is a root between (1.2,1.4).

Solution: (3): Let n=5, h=1

x	-1	0	1	2	3	4
f(x)	+	+	-	+	+	+

There is a root between (0,1) and (1,2).

It's clear that we study (in this chapter) numerical methods for solving equation : $f(x) = 0 \ldots (1)$

where x and $f(x)$ are real . The values of x for which (1) holds are called roots of equation for example :-

$x^2 - x - 2 = 0$ …(1-1)

$x^3 + x^2 - 3x - 2 = 0$ ...(1-2)

$2^x - 5x + 2 = 0$ ...(1-3)

$e^x - 3x = 0$ …(1-4)

Equation (1-1) is a second degree polynomial whose roots are $x = 2, x = -1$, but for the other equation (1-2,1-3,1-4) it is difficult (if it is not possible) to be able to express the roots through a formula as equation (1-1) , hence we have to obtain the roots of the equation(1-2,1-3,1-4) numerically we study some methods

1- False-Position method (Regula falsi method):

Suppose a continuous function f defined on the interval [a,b] is given with f(a) and f(b) of opposite sign (i.e. f(a)×f(b)<0). To derive a formula for false-position method, approximate the graph of f by a straight line on [a,b] connecting (a, f(a)) and (b, f(b)) which intersect x-axis at (c,0) where c is more approximate to the exact (actual) root than a and b.

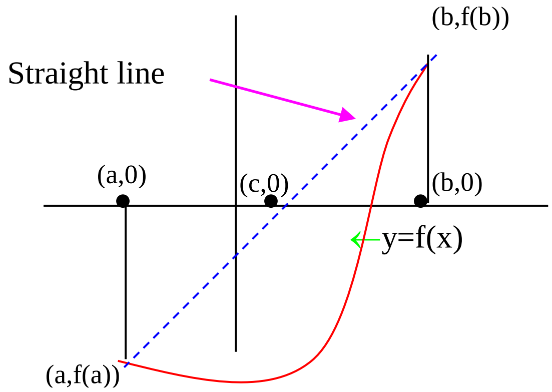
To obtain a formula for c we use the slop equality:

$$\frac{f(b)-f(a)}{b-a} = \frac{f(b)-y}{b-x} \Rightarrow \frac{f(b)-f(a)}{b-a} = \frac{f(b)-0}{b-c} \Rightarrow c = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

To find another approximation:

$$\text{If } f(a)\times f(c) \begin{cases} <0 & \text{there is a root between a, c} \Rightarrow d=\dfrac{af(c)-cf(a)}{f(c)-f(a)} \\ >0 & \text{there is a root between b, c} \Rightarrow d=\dfrac{bf(c)-cf(b)}{f(c)-f(b)} \\ =0 & \text{c is exact root ((Stop)).} \end{cases}$$

We stop iteration if the interval width is as small as desired i.e. $|x_i - x_{i+1}| < \varepsilon$ for any i.



Example :

Use an approximate root of $f(x) = xlogx - 1 = 0$ by using False position method with $\varepsilon = 0.005$ for five digit

Solution :

Let $x_1 = 2$, $x_2 = 3$

$f(x_1) = -0.39791$, $f(x_2) = 0.43136 \Longrightarrow f(x_1) \times f(x_2) < 0$

$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{2(0.43136) - 3(-0.39791)}{0.43136 - 0.39791} = 2.47985$, $\because |x_3 - x_2| > \varepsilon$

$f(x_3) = -0.02188$ find $x_4 \Longrightarrow f(x_2) \times f(x_3) < 0$

$$\Longrightarrow x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \frac{3(-0.02188) - 2.47985(0.43136)}{-0.02188 - 0.43136}$$

$= 2.50496$, $\because |x_4 - x_3| > \varepsilon$

$f(x_4) = -0.00101$ find $x_5 \Longrightarrow f(x_2) \times f(x_4) < 0$

$$\Longrightarrow x_5 = \frac{x_2 f(x_4) - x_4 f(x_2)}{f(x_4) - f(x_2)} = \frac{3(-0.00101) - 2.50496(0.43136)}{-0.00101 - 0.43136}$$

$=2.50611$

$\because |2.50611 - 2.50496| = 0.001 < \varepsilon$

$\therefore$ The root is 2.50611

H.W:

Use false position method to find the root of $x^5 - x - 0.2$,$\varepsilon = 0.005$

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